Master in Mathematical Finance

Interest Rate and Credit Risk Models

Exam – 10 January 2020

Time: 2:15h

1. Please consider the following information on the Euro area money market and Portuguese Government debt yields:

Interest rates (%)

Maturities	Euribor		Portuguese Government Debt	
	30 Dec.2019	31 Jan.2019	30 Dec.2019	31 Jan.2019
Overnight	-0,485	-0,40		
1 week	-0,480	-0,38		
1 month	-0,470	-0,37		
3 months	-0,425	-0,30		
6 months	-0,280	-0,24		
1 year	-0,269	-0,11		
2 years			-0,557	-0,09
5 years			-0,136	0,47
10 years			0,403	1,67

1.1. Compute the price of a futures contract for the 3-month Euribor, with expiry date on the 10th April 2020, for the most recent date represented. (2,0)

- Implied interest rate is approximately the 0,25f0,25

1.2. Considering that the 2 and 5 year maturities of the Portuguese Government debt are represented by bonds paying annual coupons of 0,5% and 0,75%, respectively, with a redemption value of 1000 Euros, compute the number of 5-year bonds to use in a duration hedging strategy of a portfolio comprised by 100 bonds representative of the 2-year maturity. (2,0)

1.3. Compute the 2-year spot rate using a bootstrapping methodology and identify the main conceptual differences to the yield to maturity. (1,5)

1.4. Considering the main explanatory theories of the term structure of interest rates, analyze the changes between the two dates mentioned in market expectations about the 3-month interest rates. (2,0)

- Compute 0,25f0,25 for both dates
- According to the pure or non-pure version of expectations theory, expectations increased. The remaining theories do not allow any conclusions.

1.5. Assuming that interest rate volatility is constant along time, please characterize an affine model for the term structure of interest rates in Portugal, with non-identified factors, including:

1.5.1. The observation and transition equations to be estimated. (1,25)

$$y_{n,t} = \frac{1}{n}\left(A_n + B_{1,n} z_{1t} + B_{2,n} z_{2t}\right)$$

$$\begin{bmatrix} z_{1,t+1} \\ z_{2,t+1} \end{bmatrix} = \begin{bmatrix} \varphi_1 & 0 \\ 0 & \varphi_2 \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} + \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} v_{1,t+1} \\ v_{2,t+1} \end{bmatrix}$$

1.5.2. The equation(s) necessary to assess market expectations about the future path of short-term interest rates. (1,25)

$$\begin{aligned} E_t\left(y_{1,t+n}\right) &= E_t\left(\xi - \frac{1}{2}\lambda^T \alpha \lambda + \left[\gamma^T - \frac{1}{2}\lambda^T \beta^T \lambda\right] z_{t+n}\right) \\ &= \xi - \frac{1}{2}\lambda^T \alpha \lambda + \left[\gamma^T - \frac{1}{2}\lambda^T \beta^T \lambda\right] E_t\left(z_{t+n}\right) \\ &= \xi - \frac{1}{2}\lambda^T \alpha \lambda + \left[\gamma^T - \frac{1}{2}\lambda^T \beta^T \lambda\right]\left[\left(I - \Phi^n\right)\theta + \Phi^n z_t\right] \end{aligned}$$

With β=0

2. Please consider the following information:

Average One-Year Letter Rating Migration Rates, 1920-2017

From/To	Aaa	Aa	A	Baa	Ba	B	Caa	Ca-C	WR	Default
Aaa	86.86%	7.77%	0.79%	0.19%	0.03%	0.00%	0.00%	0.00%	4.36%	0.00%
Aa	1.05%	84.11%	7.73%	0.72%	0.16%	0.05%	0.01%	0.00%	6.11%	0.06%
A	0.07%	2.70%	85.06%	5.56%	0.64%	0.12%	0.04%	0.01%	5.73%	0.08%
Baa	0.03%	0.23%	4.20%	82.90%	4.55%	0.72%	0.13%	0.02%	6.96%	0.25%
Ba	0.01%	0.07%	0.49%	6.15%	74.05%	6.85%	0.67%	0.09%	10.49%	1.14%
B	0.00%	0.04%	0.16%	0.61%	5.56%	71.76%	6.19%	0.46%	12.00%	3.21%
Caa	0.00%	0.01%	0.03%	0.11%	0.51%	6.71%	67.72%	2.95%	13.90%	8.06%
Ca-C	0.00%	0.02%	0.10%	0.04%	0.57%	2.82%	8.19%	47.69%	18.45%	22.13%

Source: Moody's (2018), Corporate Default and Recovery Rates, 1920-2017".

2.1. According to the information above, calculate the annual premium for a credit default (digital) swap with the following data: (4,0)

- Underlying asset - bonds issued by company FinCorp, with a Baa rating
- Maturity = 2 years
- Notional amount = €100000
- CDS fee = s%
- Frequency of swap payments = yearly
- Default payment = 65%
- Defaults assumed to occur at the swap payment dates (if a default occurs)
- Risk-free interest rate = 0,5% (continuously compounded, flat yield curve)

2.2. Calculate the 2-year cumulative probability of default, assuming that the annual hazard rate for the same Baa rating changes to 0,5% in the second year and new information arriving during the first period may generate two survival scenarios for the second year, being 0,7 the probability of the most likely scenario. (3,0)

$$\mathrm{P}(2) = \mathrm{P}(1) \cdot p(2|1) = e^{-\lambda(1)} \cdot E\left[e^{-\lambda(2)}\right] = E\left[e^{-[\lambda(1)+\lambda(2)]}\right]$$
$$p(2|1) = qe^{-\lambda(2,H)} + (1-q)e^{-\lambda(2,L)} = E\left[e^{-\lambda(2)}\right]$$

2.3. What could you conclude from a sharp decrease in the share prices of FinCorp regarding its credit risk? In your answer, please characterize the methodology used to reach your conclusions, including the information you would need to present a quantified measure of credit risk based on that methodology, as well as its limitations. (3,0)

- Depends on the behaviour of other variables, e.g. MVA and liabilities
- Ceteris paribus, this may be interpreted as suggesting a PD increase, according to Merton Model.