Master in Mathematical Finance

Interest Rate and Credit Risk Models

Exam – 5 February 2020

Time: 2:15h

1. Assuming that you are assessing the behaviour of Government bond markets in Portugal and estimated the yield curve with the Svensson methodology, having obtained the following estimates for the parameters β_0 and β_1 in two different dates:

	4 Feb.2020	4 Feb.2019
β_0	0.005	0.02
β_1	-0.005	-0.022

1.1. Interpret the main changes occurred in the yield curve between the two dates, considering the different explanatory theories of the term structure of interest rates. (3,0)

- Decrease in long term interest rate
- Decrease in the slope
- Expectations theory – lower short-term interest rate expectations
- Liquidity preference theory – reduction in the term premium
- Segmentation theory – lower demand for long-term bonds
- Preferred habitat theory – lower demand for long-term bonds and/or lower liquidity premium

1.2. Which bond investment or hedging strategy would you implement to manage interest rate risk, according to the figures provided? (2,0)

- as short-term interest rates are expected to increase but only slightly, no hedging strategy could be necessary

- but if lower long-term interest rates are not reflecting expectations of lower short-term interest rates, we could need to reduce the duration by selling futures or decreasing the weight of bonds with higher durations.

1.3. Compare the main features of this methodology, including pros and cons, to 2 alternative static methodologies to estimate the yield curve. (3,0)

- easy estimation

- economic meaning of (some) parameters

- ensures consistent forward (and spot) curves

1.4. What are the impacts on the forecast of future values for short-term interest rates of assuming that these interest rates follow a Wiener or a Generalized Wiener process? (2,0)

- A stochastic process z follows a **Wiener process (or the continuous random walk) => Δz=0 => uncertainty is proportional to the square root of time and the Wiener process follows a normal distribution with drift = 0 => the expected value of any future outcome is equal to the current value:**
- Generalized Wiener process **- The average increases are proportional to time**
- **Consequently, if one assumes that interest rates follow one of these stochastic processes, the forward rates cannot be used as proxies for the future interest rates.**

2. Please assume that one year ago you have invested in bonds issued at par by company NFC based in Portugal:

	04.02.2020	04.02.2019
Number of shares issued (Millions)	5	5
Nominal value of shares (€)	10	10
Market Value of shares (€)	8	11
Annualized volatility of share prices (%)	60	30
Short-term Liabilities (Million €)	70	40
Long-term Liabilities (Million €)	30	20
Annualized volatility of Market Value of Assets (%)	20	25
Market Value of Assets (Million €)	100	130
1-year interest rate (%)	0,05	0,1

2.1. Using the Merton model, assess the changes in the credit risk of NFC, assuming a drift of 20%/year, detailing your calculations and presenting any additional assumption you may be required to consider. (5,0)

2.2. In order to mitigate the credit risk in this investment, please describe how could you use a 2-year credit default swap and calculate the annual premium assuming that:

(i) the payment in case of default corresponds to 75% of the par value;
(ii) the yield curve is completely flat;
(iii) the marginal probabilities of default (PD) double every year since 1-year maturity. (2,5)

Notes: If necessary, please assume a 1-year PD of 2% and state any additional assumption you may have decided to take.

2.3. Assuming that the default intensity follows a Poisson process, what would be the default intensity necessary to get the 1-year default rate for NFC at the same moment considered in the previous question and the corresponding expected time to default? (2,5)

$p(t) = e^{-\lambda t}$ - probability of survival for t years

$1/\lambda$ - expected time to default

1-0,002 = = $e^{-\lambda t}$ ⇔ ln(1-0,002) = $-\lambda t$ ⇔ λ=-ln(1-0,002)/t => λ=-ln(1-0,002) =0,213% (as t=1)

$1/\lambda$ = 496 years