Master in Mathematical Finance

Interest Rate and Credit Risk Models

Exam – 19 January 2021

Time: 2:15h

1. Considering the estimates for the parameters β_0 and β_1 in the Svensson methodology and two different dates for the Portuguese yield curve:

	4 Jan.2021	4 Jan.2020
β_0	0.002	0.005
β_1	-0.043	-0.022

1.1. Interpret the main changes occurred in the yield curve between the two dates, considering the different explanatory theories of the term structure of interest rates. (2,5/20)

- Interpretation of the parameters
- Main changes:
  - Non-parallel downward shift of the yield curve, with an increase in the slope of the yield curve

- Interpretation of the curve shifts:

- expectations theory - reduction of expected future short-term interest rates, namely in the next few years

- liquidity preference theory – increase in the long-term liquidity premium

- preferred habitat theory – a mix of the former two

- segmentation theory – changes result from the interaction of supply and demand in each maturity bucket, e.g. impact of monetary policy decisions on the short-end of the curve and increasing risk-aversion in the markets, leading to an increase in the demand of long-term govt bonds.

1.2. Assuming that:

- most shifts in this yield curve are parallel or just involve changes in the slope;
- the volatility of interest rates is constant;

present an adequate affine model to characterize the behavior and the shape of the curve along time, by specifying the main equations (e.g. the spot, the one-period forward, the volatility and the term premium curves). (2,5/20)

- Main equations of the 2-factor Vasicek model – equations 84, 88, 89 and 90

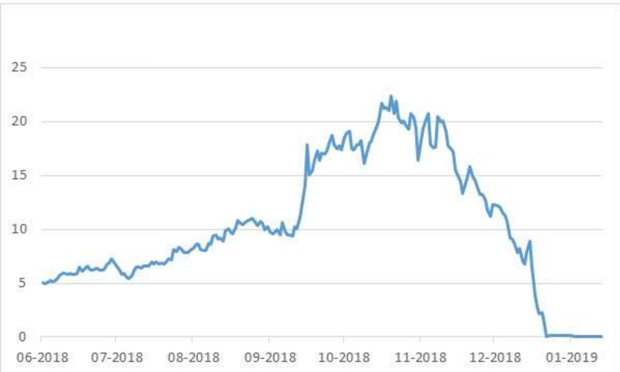
1.3. Explain how could you obtain an estimate for the 3-month Euribor rate at a given future date considering the models employed in the two previous questions. (2,5/20)

- Estimate the yield curve and compute the forward rate from the 3-month spot rate and the spot rate for a maturity corresponding to 3 months + time to settlement, f(m+n) = [(1+s(m+n))^(m+n)/(1+s(m))^(m)]^(1/n)-1, i.e. assuming that the pure version of expectations theory holds.

1.4 Explain the main differences in the informational content about this future value of the 3-month Euribor rate if you use futures and option prices. comparing to the previous questions. (2,5/20)

- Expected value vs the full RND

2. Please consider the following information about the stock prices of FinCorp Bank, as well as the marginal probabilities of default at the initial date in the chart:



Maturity (years)	PD
1	1,0%
2	2,0%

2.1. Compute the premium of a credit default swap for that initial date, with the following features and assuming that interest rates are zero for all relevant maturities: (2,5/20)

Maturity = 2 years
Notional = € 20.000.000
Payment in case of default = 70% of the notional

Answer: Binomial tree in the slides or the computation in excel

Binomial tree:

l E_1[*pay-off*(2)|ND] = *pay-off* in case of no default *(1-PD) + *pay-off* in default *PD =0 x (1– 0.02) + (1-0.7) x 0.02 = 0.014

E_1[*pay-off*(2)|D] = 0.7
E_0[X(2)] = E_1[*pay-off*(2)|ND]*(1-PD) + E_1[*pay-off*(2)|D]*PD = 0.014 x (1–0.01) + 0.7 x 0.01 = 0.0198

2.2. Consider the following information about the 1-year rating transition matrix available from the statistics of a recognized rating agency at the same date:

	I	S	D
I	0,90	0,09	0,01
S	0,10	0,85	0,05

2.2.1. Assuming that the default intensity follows a Poisson process and that FinCorp Bank had a rating of "I", what would be the default intensity necessary to get the 1-year default rate at that date and the corresponding expected time to default? (2,5/20)

- See excel file

2.2.2. Calculate the 2-year cumulative probability of default, assuming that the annual hazard rate for the same rating changes to 1,5% in the second year and new information arriving during the first period may generate two survival scenarios for the second year, being 0,8 the probability of the most likely scenario. (2,5/20)

$$\mathrm{P}(2) = \mathrm{P}(1) \cdot p(2|1) = e^{-\lambda(1)} \cdot E\left[e^{-\lambda(2)}\right] = E\left[e^{-[\lambda(1)+\lambda(2)]}\right]$$
$$p(2|1) = qe^{-\lambda(2,H)} + (1-q)e^{-\lambda(2,L)} = E\left[e^{-\lambda(2)}\right]$$

2y cumulative PD = 1 – P(2)

2.3. Interpret the behavior of the probability of default according to the chart by using a structural model, explaining how would you estimate it and presenting the main assumptions, equations and limitations of the model. (2,5/20)

- Merton Model
- Stock price – price of a call-option on the market value of assets, with the strike price being the redemption value of debt
- PD = probability of MVA > Debt
- Model equations
- The stock price suggests an approximation to a default status