Master in Mathematical Financial

Interest Rates and Credit Risk Models

Exam – 20 January 2022

1. Considering a bank portfolio of homogeneous corporate loans, comprising 100 contracts, each with a value of 1 Million € and a 5-year maturity, compute and interpret the results of the following questions:

1.1. The 1-year Credit-VaR at 99% level of confidence and the capital requirement for this portfolio, under the Gaussian Copula hypothesis and taking into account the following data obtained from internal models: **(4,0/20)**

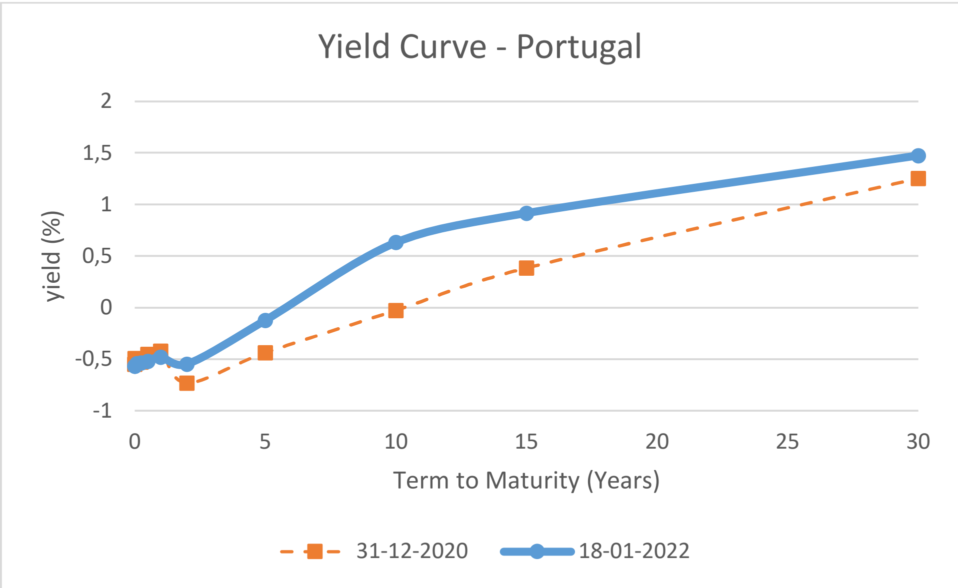
1y PD = 2%
Correlation coefficient (ρ) = 0.01
Recovery Rate (RR) = 40%

1.2. The minimum number of contracts to get into default, in order to obtain a credit loss corresponding to the Credit-VaR computed in the previous question, and the probability of getting that number of contracts into default, assuming they are independent. **(2,5/20)**

- similar figures, due to the fact that the correl.coefficient in 1.1. is close to zero.

1.3. The premium of a 2-year credit default swap on the loan portfolio, assuming that interest rates are zero for all relevant maturities, the hazard rate corresponds to the 1y PD in question 1.1. and all potential defaults occur at the end of a year. **(3,5/20)**

2. Consider the following information about the evolution of the Portuguese yield curve, where Euro money market rates are used for maturities up to 1 year and yields-to-maturity of fixed rate Portuguese Government bonds are considered for maturities beyond:



2.1. Characterize the main factors behind the changes observed in the yield curve, according to the explanatory theories of the term structure of interest rates, the main drawbacks of using this information to characterize the term structure of interest rates and the alternative ways to characterize it. **(2,5/20)**

- expectations theory – increase in expectations, with the implied forwards corresponding to the expected values in the pure version and the the variation corresponding to the change in expectations, in non-pure version.

- segmentation and preferred habitat theory – no relationship between different rates

- liquidity theory – increased term premium

- different ways to characterize the term structure of interest rates – spot, forward and discount curves

2.2. Describe the main pros and cons of using the Nelson-Siegel, Svensson and McCulloch methods to fit the yield curve, identifying the method you would choose and the main changes in the parameters between the two curves if you had opted for the Nelson-Siegel method. **(2,5/20)**

- Being a polynomial method, McCullhoch provides better in-sample fitting. However, the yield curve is unstable out of sample and tend to provide meaningless forward rates.

- NS and Svensson provide yield curve with more consistent shapes and economic meaning of the parameters, even though with worse in-sample fitting.

- Svensson is better whenever the curve has more than one inflection point, like the more recent curve

2.3. Explain how would you fit the yield curve along time, by using:

2.3.1. A dynamic version of the Svensson method **(2,0/20)**

$$s_{t,m} = \beta_{0,\mathrm{t}} + \beta_{1,\mathrm{t}} \cdot \left[1 - e^{(-\lambda_{1t} m)}\right]/(\lambda_{1t} m) + \beta_2 \cdot \left\{\left[1 - e^{(-\lambda_{1t} m)}\right]/(\lambda_{1t} m) - e^{(-\lambda_{1t} m)}\right\}$$
$$+\beta_3 \cdot \left\{\left[1 - e^{(-\lambda_{2t} m)}\right]/(\lambda_{2t} m) - e^{(-\lambda_{2t} m)}\right\}$$

being $1/\tau_1$ by λ_1 and $1/\tau_2$ by λ_2

2.3.2. An affine model, assuming that the curve is mostly driven by two factors, one of them linked to the inflation rate and the volatility is constant, presenting the yield curve and the corresponding factor loading equations, as well as the model in a state-space form. **(3,0/20)**

$$y_{n,t} = \frac{1}{n}\left(A_n + B_{1,n} z_{1t} + B_{2,n} z_{2t}\right)$$

$$A_n = A_{n-1} + \delta + \frac{1}{2}\sum_{i=1}^{k}\left[\lambda_i^2 \sigma_i^2 - \left(\lambda_i \sigma_i + B_{i,n-1}\sigma_i\right)^2\right]$$

$$B_{i,n} = \left(1 + B_{i,n-1}\varphi_i\right)$$

$$\begin{bmatrix} y_{1,t} \\ \vdots \\ y_{l,t} \\ Var_t(y_{1,t+1}) \\ \vdots \\ Var_t(y_{l,t+1}) \\ \tilde{\pi}_t \end{bmatrix} = \begin{bmatrix} a_{1,t} \\ \vdots \\ a_{l,t} \\ a_{l+1,t} \\ \vdots \\ a_{2l} \\ 0 \end{bmatrix} + \begin{bmatrix} b_{1,1} & b_{2,1} \\ \vdots & \vdots \\ b_{1,l} & b_{2,l} \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & b_{\pi} \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} + \begin{bmatrix} \upsilon_{1,t} \\ \vdots \\ \upsilon_{l,t} \\ \upsilon_{l+1,t} \\ \vdots \\ \upsilon_{2l,t} \\ \upsilon_{\pi} \end{bmatrix}$$