Master in Mathematical Financial

Interest Rates and Credit Risk Models

Exam – 3 February 2022

1. Considering the following information provided from the estimation of the German Yield Curve using the Svensson static methodology:

1.1. Characterize the parameters in the table and the main changes in the yield curve between the two dates, interpreting the meaning of these changes according to the different explanatory theories of the term structure of interest rates and proposing a hedging initiative to the holder of long-term bonds issued by the German Government . **(2,5)**

	1 Feb.2022	14 Jan.2022
β_0	0.41	0.32
β_1	-0.96	-0.85

- increase in the long term rate, leading to an increase in the slope of the curve, illustrating expectations of higher short-term rates in the future, according to the expectations theory, or an increase in liquidity premium, according to the liquidity preference theory
- hedging: decrease duration

1.2. Using the information below for the German spot curve at the most recent date stated in the previous question, compute the price of a futures contract on the 3-month interest rate with a settlement date 1 year from that date. **(2,5)**

Maturity (Years)	Spot Rate
0,003	-0,5462
0,25	-0,5386
0,50	-0,5290
0,75	-0,5178
1,00	-0,5051
1,25	-0,4913
1,50	-0,4765
1,75	-0,4608
2,00	-0,4445

Futures price = 100 – implied forward

$$ {}_{m}f_{n} = \left[\frac{(1+s_{m+n})^{m+n}}{(1+s_{m})^{m}} \right]^{\frac{1}{n}} - 1 $$

$$ {}_{1}f_{0,25} = \left[\frac{(1+s_{1,25})^{1,25}}{(1+s_{1})^{1}} \right]^{1/0,25} - 1 = -0,00436 $$

2. Considering a financial asset price following a stochastic process that corresponds to a Geometric Brownian Motion, with a drift of 0,3 and a volatility of 25% (both per annum):

2.1. Characterize the distribution of the growth rate of the asset price, including the calculation of the expected value and the variance of this growth rate for a period of one month. **(2,5)**

See excel

2.2. Present the stochastic process of the forward rate on this asset price. **(3,0)**

See excel

2.3. Compute an estimate for this asset price one month from today, being the current price equal to 130 and the random component equal to zero. **(3,0)**

See excel

2.4. What would be the consequences for the stochastic process if shorter periods of time were considered in the previous questions? **(1,5)**

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- When Dt -> 0, the path becomes much more irregular, as the size of the movement in the variable in time Dt is proportional to the sqrt(delta(t)). When Dt is small, the sqrt(delta(t)) is much larger than Delta(t) => the changes in z will be much larger than Dt, as $\Delta z = \epsilon\sqrt{\Delta t}$

3. Consider the following information about a 1-year rating transition matrix available from the statistics of a recognized rating agency:

	I	S	D
I	0,85	0,13	0,02
S	0,20	0,70	0,10

3.1. Assuming that the default intensity follows a Poisson process and that a given debt issuer had a rating of "I", what would be the default intensity necessary to get the 1-year default rate at that date and the corresponding expected time to default? **(2,5)**

See excel file

3.2. Calculate the 2-year cumulative probability of default, assuming that the annual hazard rate for the same rating changes to 3% in the second year and new information arriving during the first period may generate two survival scenarios for the second year, being 0,75 the probability of the most likely scenario. **(2,5)**

See excel file