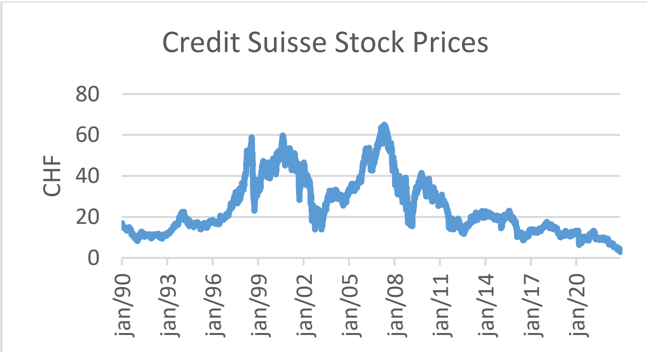
ISEG Lisbon School of Economics & Management
Universidade de Lisboa

Master in Mathematical Financial

Interest Rates and Credit Risk Models
Exam – 6 January 2023

1. Considering the information below about Credit Suisse stock prices and rating transitions according to Moody's:

1.1. Explain what could you conclude about the credit risk evolution of this bank, using a structural model, detailing the key features of that model. **(3,0/20)**



- PD must be increasing, ceteris paribus.
- Key features of the Merton Model, namely assumptions, the equation system and the PD computation

1.2. Calculate the annual premium for a credit default swap with the following data: **(4,0/20)**

- Underlying asset - bonds issued by Credit Suisse, assuming a Ba rating
- Maturity = 2 years
- Notional amount = € 1 000 000
- Frequency of swap payments = yearly
- Default payment = 75%
- Defaults assumed to occur at the swap payment dates (if a default occurs)
- Risk-free interest rate = 3% (continuously compounded, flat yield curve)

Average one-year letter rating migration rates, 1920-2021

	Aaa	Aa	A	Baa	Ba	B	Caa	Ca-C	WR	Def
Aaa	87.05%	7.60%	0.77%	0.18%	0.03%	0.00%	0.00%	0.00%	4.37%	0.00%
Aa	0.99%	84.41%	7.64%	0.69%	0.15%	0.04%	0.01%	0.00%	6.00%	0.06%
A	0.06%	2.66%	85.66%	5.28%	0.58%	0.11%	0.03%	0.01%	5.55%	0.08%
Baa	0.03%	0.21%	3.93%	83.90%	4.21%	0.67%	0.11%	0.02%	6.70%	0.22%
Ba	0.01%	0.07%	0.46%	6.05%	74.50%	6.73%	0.66%	0.09%	10.38%	1.07%
B	0.00%	0.04%	0.14%	0.58%	5.47%	72.15%	6.31%	0.46%	11.86%	3.00%
Caa	0.00%	0.01%	0.02%	0.09%	0.39%	6.09%	69.28%	2.89%	14.02%	7.22%
Ca-C	0.00%	0.01%	0.08%	0.03%	0.43%	2.54%	9.23%	44.88%	17.41%	25.39%

Source: Moody's (2022), Corporate Default and Recovery Rates, 1920-2021".

- See excel

1.3. What is the default intensity corresponding to the 1-year default rate of the bank and its expected time to default, assuming the default intensity follows a Poisson process? **(2,5/20)**

- See excel

2. Considering the following information provided from the estimation of the Portuguese Yield Curve using the Nelson and Siegel static methodology:

2.1. Characterize the parameters in the table, the main changes in the yield curve between these dates and the potential advantages of using alternatively the Svensson methodology. **(2,5/20)**

	5 Dec.2022	5 Dec.2021
β_0	3,5%	1,25%
β_1	-1,5%	-1,25%

$\beta 0$ – asymptotic (long-term) interest rate increased, following monetary policy tightening

$\beta 0$+ $\beta 1$ – instantaneous (short-term) interest rate increased, following monetary policy tightening too

–$\beta 1$ – slope increased from 1,25% to 1,5%

2.2. Interpret those changes according to the different explanatory theories of the term structure of interest rates and propose an investment or hedging strategy to be adopted by a manager of a portfolio comprising Portuguese Government Bonds. **(3,0/20)**

- Expectations theory (pure version) – expectations about short-term interest rates increased
- Expectations theory (non-pure version) – expectations about short-term interest rates or term premium increased
- Liquidity preference - term premium increased
- Preferred habitat - term premium increased to convince investors preferring shorter maturities to buy longer-term bonds
- Segmentation – long-term interest rates increase (or price decrease) was due to lower demand or higher long-term bonds offer.

  If one expects interest rates to move higher, the option should be to decrease the average duration of the portfolio, namely by selling futures contracts or selling longer duration bonds, using those proceeds to buy shorter duration bonds, always preferring bonds with higher convexity.

2.3. Using the information below for the Euribor rates at the most recent date mentioned in question 2.1, compute the price of a futures contract on the 3-month Euribor interest rate with a settlement date on the 5th Mar. 2023 **(2,5/20)**

Maturity (Months)	Euribor (%)
1	1,514
3	1,975
6	2,406
12	2,811

- See excel
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2.4. Please discuss in which extent Government bonds are riskier when their pay-offs are negatively correlated to the stochastic discount factor and how could their risk be related to the marginal utility of consumption **(2,5/20)**

$$\Lambda_t = E_t\left[i_{t+1}\right] - i_{t+1}^f = -\rho_{M_{t+1},i_{t+1}} \frac{\sigma_{M_{t+1}} \sigma_{i_{t+1}}}{E_t\left[M_{t+1}\right]}$$

- Explain CCAPM => discount rate as the intertemporal marginal rate of substitution =>

  in equilibrium the price of a financial asset is such that the utility loss from not consuming the income allocated to the financial asset corresponds to the discounted expected utility from consuming the pay-off of the financial asset in the next period.