Instituto Superior de Economia e Gestão

Decision Making and Optimization

Final Exam with Solutions 2023-2024

January 11, 2024 **Duration: 2h**

Notes: -Justify all answers and present the calculations carried out.
-Answer all questions using methodologies taught in Decision Making and Optimization classes.

Name: ______________________ **No.** ________

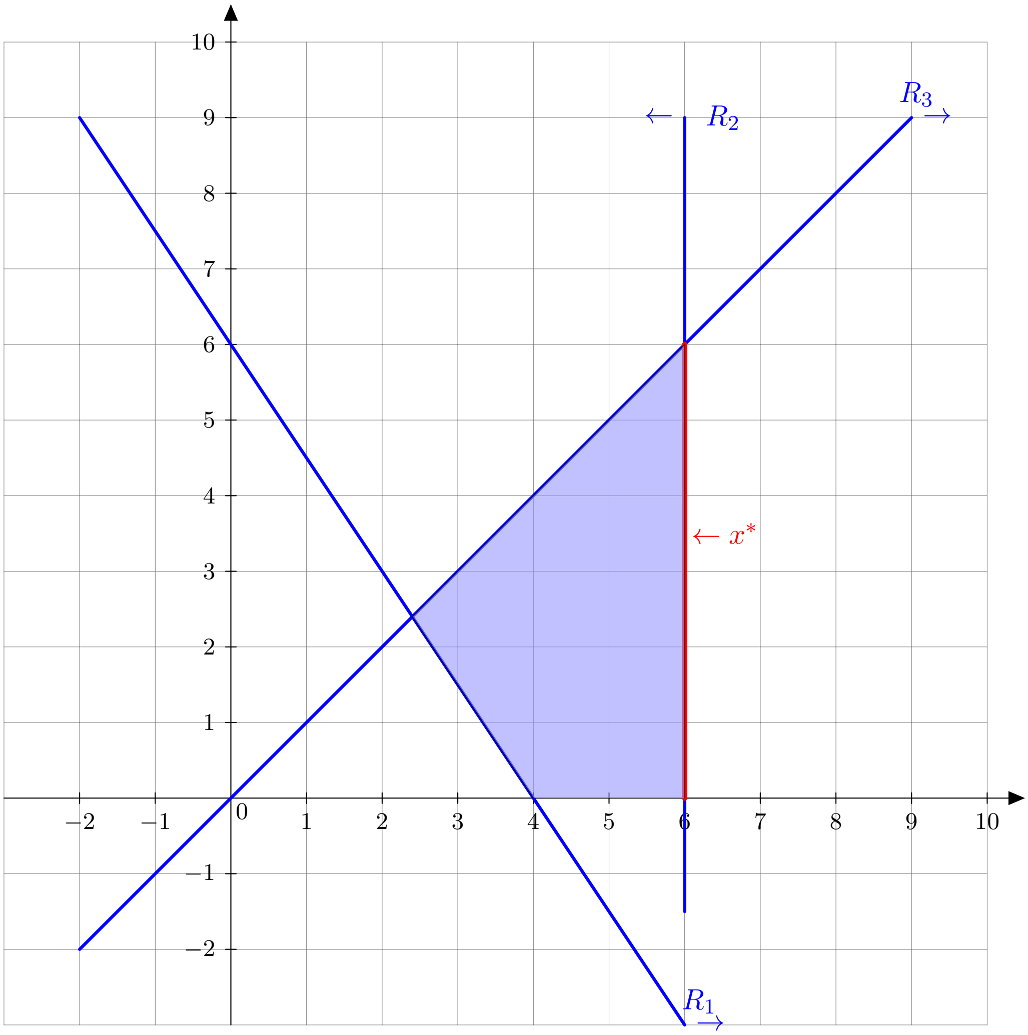
This test has 5 pages.
Write your answers here.
Good job!

Question	1	2	3	4	5	total
Quotation	25	75	30	30	40	200
Classification						

1. Consider the following linear programming problem.

$$
\begin{aligned}
(P_1)\ \max\quad & 3x_1 \\
\text{s.t.}\quad & 3x_1 + 2x_2 \geq 12 \\
& x_1 \leq 6 \\
& x_1 - x_2 \geq 0 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

(a) Use the graphical method to solve problem (P_1).



Write here all the optimal solutions:

$$x^* = \alpha(6,0) + (1-\alpha)(6,4),\ \alpha \in [0,1]$$

(b) Take the two points $(2,0)$ and $(4,0)$ and classify the corresponding two solutions in feasible or non feasible and in basic or non basic.

point $(2,0)$ corresponds to the solution $(2,0,-6,4,2)$ which is a non feasible and non basic solution

point $(4,0)$ corresponds to the solution $(4,0,0,2,4)$ which is a feasible and basic solution

2. The operations research department of a company wishing to optimise the profit from the production of two products P_1, P_2, using two limited resources R_1, R_2, solves the LP problem shown on the left, and its optimal simplex table is shown on the right.

$$\begin{aligned} \max \quad z = \ & 200x_1 + 300x_2 \\ \text{s.a} \quad & 2x_1 + 5x_2 \leq 180 \\ & 3x_1 + 3x_2 \leq 135 \\ & x_1, x_2 \geq 0 \end{aligned}$$

x_B	x_1	x_2	x_3	x_4	$\bar{b}$
x_1	1	0	$-1/3$	$5/9$	15
x_2	0	1	$1/3$	$-2/9$	30
$z_j - c_j$	0	0	$100/3$	$400/9$	12000

The Solver Excel Sensitivity Report is displayed below.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$5		15	0	200	100	80
\$C\$5		30	0	300	200	100

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$D\$2		180	33.33333333	180	45	90
\$D\$3		135	44.44444444	135	135	27

(a) Write and interpret the optimal production plan (include the value of the objective function and the slack (or auxiliary) variables).

$x^* = (15, 30, 0, 0, 0),\ z^* = 12000$

(b) Write the dual and interpret the meaning of the first dual variable.

$$\begin{aligned}(D)\ \ \min \quad & 180y_1 + 135y_2 \\ \text{s.t.} \quad & 2y_1 + 3y_2 \geq 200 \\ & 5y_1 + 3y3 \geq 300 \\ & y_1, y_2 \geq 0\end{aligned}$$

(c) What profit for product P_2 will lead to no production of product P_1? What will the new optimal production values be?

$\bar{c}_2 \geq 500, \qquad \bar{x}^* = (0, 36, 0, 37)$

(d) What will be the effect on the optimal solution if the available resource R_2 is increased to 165.

no change in the production plan, P_1, P_2 will be produced,

but the values will change $\bar{x}^* = (31, (6); 36, (6))$

(e) What is the maximum price the company would be willing to pay for additional resource $R2$. How many additional resource could be purchased at that price?

maximum price $44, (4)$,
maximum purchase 135

(f) The company accepted to purchase additional resources. But only one of the two resources will be purchased. Which one should it be? Why?

R_2

3. Consider the following problem faced by a company with three supply centres and four customers. A commodity has to be transported from the suppliers to the customers and the company wants to minimize the transportation costs. The following table shows the unit transport costs (in $\$10^3$ per tonne transported) of the commodity, the supply at the origins and the demand at the destinations.

	1	2	3	4	Supply
1	2	4	6	9	20
2	7	5	2	3	40
3	9	11	7	8	50
Demand	10	40	30	40	

(a) Identify, among the problems studied, one that could be used to solve this problem and present the Linear Programming formulation of the problem.

Transportation problem with unbalanced supply and demand

(b) Propose a basic feasible solution to the problem. Explain how you arrived at the proposed solution.

$x^* = (10, 10, 0, 0, 0, 30, 10, 0, 0, 0, 20, 30)$

4. Consider the Binary Knapsack Problem and the following instance with $n = 7$ objects, capacity $C = 9$, utilities $u = (7, 2, 4, 3, 5, 8, 3)$ and volumes $v = (3, 6, 2, 3, 2, 2, 1)$. Propose a lower and an upper bound for the optimal value of the problem by using the critical index. Explain how you arrived at the proposed bounds.

critical index $k = 5$

$LB = 23$

$UB = 25$

5. An individual has 15000€ at his disposal and has to decide whether to invest in a certain product. He believes that there is a 65% chance that the economy will be favourable to the investment and expects to double the amount invested. However, he would lose 4000€ if the economy is not favourable. Before making a decision, he can ask an expert for advice at a cost of 1250€.

   (a) Build the payoff matrix.

	favourable	non favourable
invest	30000	11000
do not invest	15000	15000
a priori probabilities	0,65	0,35

   (b) What is the decision recommended by the Laplace criteria and explain its meaning.

   Invest

   (c) What is the decision recommended by the Bayes criteria.

   Invest

   (d) Calculate the EVPI (expected value of perfect information) and on the basis of this value say what you advise.

   $EVPI = 1400$