Lisbon School of Economics & Management
Universidade de Lisboa

Microeconomics
Fall 2024-2025
Regular exam
December 2024

Duration: 3 hours (180 minutes)

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General Guidelines

- You may use a calculator;
- You may **not** use a programmable calculator;
- You may **not** use notes or books;
- You may have some food and beverages on your desk;
- All other belongings, including phones, must be on the floor;
- You can only leave the room after 30 minutes into the exam and up unto 15 minutes before the exam ends;
- Write all your answers on the blank answer sheets brought by you;
- Write your name and student number on every answer sheet;
- Number all your answer sheets and hand them in in chronological order;
- If a question does not ask for an explanation, there is no need to give one;
- This exam is to be handed in together with your answer sheets;
- Any form of fraud will, at least, imply an invalid grade for this course.

1. Production (3 points)

Let $y = \theta + \beta x_1 + \gamma x_2$ be a production function, where y is the output and x_1 and x_2 are the two inputs.

1.1. Find the Technical Rate of Substitution (TRS) for the production above.

Consider for the following two questions that $\theta = 0$, and $\beta = 2$ and $\gamma = 2$.

1.2. Carefully sketch the input requirement set for producing at least 10 units of output:

$$\{ (x_1, x_2) \;\; in \; R_+^2 \;\mid\; 2x_1 + 2x_2 \geq 10 \}$$

1.3. Consider that in the short run x_2 is fixed at a value of 2. Carefully sketch the short-run production possibilities set:

$$\{ (y, x_1) \;\; in \; R_+^2 \;\mid 2x_1 + 2x_2 \geq y, x_2 = 2 \}$$

2. Profit and costs (4 points)

2.1. Consider a firm's (optimized) profit function $\pi(p, w, x(p, w))$, where $x(p, w)$ is the optimally chosen input x as a function of p and w, which are the price of the output and input respectively. Take the derivative of this profit function towards p and use the envelope theorem to discuss how a firm's profit changes when p changes.

Consider for the following three questions that the firm uses two inputs x_1 and x_2 to produce output y via the following production function: $y = 1x_1 + 1x_2$. Consider that the price for inputs x_1 and x_2 are $w_1 = 1$ and $w_2 = 2$ respectively.

2.2. Carefully sketch the isoquant that allows the firm to produce exactly 5 units of output:

$$\{ (x_1, x_2) \;\; in \; R_+^2 \;\mid 1x_1 + 1x_2 = 5 \}$$

Carefully sketch into the same graph the isocost line that corresponds to the minimum costs c that allows the firm to produce those 5 units of output:

$$\{ (x_1, x_2) \;\; in \; R_+^2 \;\mid 1x_1 + 2x_2 = c \;\}$$

Conclude from your graph what are the conditional factor demands for input 1 and 2 and what are the minimum costs c to produce 5 units of output.

2.3. Now consider that the firm has the objective of maximizing its output given that its costs are equal to 10. What are the demands for input 1 and 2?

2.4. From your answers to question 2.2 and 2.3, what do you conclude about the income effect for input x_2 in case the problem was framed as a *consumer choice* problem instead? Provide a brief explanation for your answer.

3. Consumer choice (5 points)

3.1. Which assumption on consumer preferences guarantees that the Hicksian demand curve is downwards sloping. Briefly explain your answer.

Consider that the consumer has a utility function equal to $u = x_1^{\alpha} x_2^{1-\alpha}$. The consumer has income m, and the price for good x_1 and x_2 are p_1 and p_2 respectively.

3.2. Find the Marshallian demand functions for both x_1 and x_2.

3.3. Under which restriction on α are both x_1 and x_2 derived in question 3.2 ordinary goods?

3.4. Find the indirect utility function.

3.5. To find the Lagrange multiplier lambda one can take the derivative of the indirect utility function towards an exogenous variable. Which exogenous variable is this? Find the Lagrange multiplier via this route. Provide a brief economic interpretation for lambda while assuming that $\alpha = 1/2$, $p_1 = 1$, $p_2 = 4$, and $m = 10$.

4. Welfare (4 points)

Consider a consumer with a utility function equal to $u = \sqrt{x_1} + x_2$. The consumer has income $m = 50$, and the price for good x_1 and x_2 are denoted by p_1 and p_2 respectively.

4.1. Find both the Marshallian and the Hicksian demand function for good x_1.

4.2. Use the derived demand functions in question 4.1 to argue that the change in consumer surplus can be used as an exact measure of welfare.

4.3. Consider that $p_2 = 4$. Find either the change in consumer surplus, equivalent variation, or compensating variation for a change in p_1 from 1 to 2.

5. Perfect competition (2 points)

Consider a perfect competitive market. Let the total cost function of *a single* firm be equal to:

$$c(y) = 0.5y^2 + 2$$

Where *y* is the output. Let the *market* demand be given by:

$$X(p) = 40 - 2p$$

Where *p* is the price. Suppose that in the long run there is free entry into, and exit out of, this market, and that all potential firms have the same cost function $c(y)$ as above.

5.1. How many firms will there be active in this perfect competitive market in the long run?

6. Monopoly (2 points)

Consider a monopoly with the following demand function and total cost function:

$$Q(p) = 100 - 2p$$
$$c(Q) = 50 + 5Q + Q^2$$

Where Q is the quantity and p is the price.

6.1. What is the monopolist's profit-maximizing level of production and profit?

Above we considered that the monopolist charges the *same* price to *each* consumer. Now consider a monopolist that can charge a *different* price to *each* consumer. This is called perfect price discrimination.

6.2. Briefly discuss what would be the profit-maximizing pricing strategy of a monopolist that can perfectly price discriminate. Next, briefly discuss whether the *consumers* and/or *monopolist* benefit from perfect price discrimination. Finally, give a real-world example of a firm practicing (perfect) price discrimination.