A model for the analysis of monetary policy with inflation targeting

Lecture 13

Readings

- C. Walsh, “Teaching Inflation Targeting: An Analysis for Intermediate Macro”, *Journal of Economic Education,* 2002

Plan

- PC-MPR model
- Optimal monetary policy as a loss-minimization problem
- Short and long run equilibrium in the PC-MPR model
- Effects of a reduction in the inflation target
- Effect of an increase in expected inflation
- Preferences of CB and supply shocks

Preview of the PC-MPR model

- Two components:

  1. Phillips curve (PC)
  2. Monetary policy rule (MPR)
  3. (Dynamic IS curve in the back)

- Phillips curve

  – Relate current inflation to expected inflation and the output gap
  – Supply side of the model

- Monetary policy rule

  – Reflect CB preferences in trading off fluctuations in output and inflation
  – Demand side of the model (together with dynamic IS curve)

Output gap and natural level of output

- Output gap (denoted x): deviation of the current level of output from the natural level of output

- Natural level of output (denoted y^n): the level of output with perfectly flexible prices (and wages)

- Monetary policy does not affect the natural level of output

Phillips curve

$$\pi = \pi^e + \lambda x + v$$

- π: inflation rate
- π^e: expected inflation for next period on the basis of the information available today
- x: output gap
- v: inflationary shock (or supply or cost-push), capturing any other factor affecting inflation
- $\lambda > 0$: measures the slope of the Phillips curve in the plan (π, x)

Phillips curve, cont.

- Slope inversely related to the degree of rigidity in prices (and/or wages)
- The higher is price rigidity, the lower is the slope, and the lower are the effects of a given change in the output gap on inflation or, equivalently, the higher is the change in the output gap needed to achieve a given change in inflation

Phillips curve, cont.

- Presence of inflation expectations: prices are not perfectly flexible, and are adjusted based on expectations about future prices, costs, and demand conditions

- The inflation expectation term may represent expectations of current inflation (as in the simple sticky price and sticky wage model we have studied) or expectations of future inflation (as would be implied by a model where firms set prices and /or wages that remain fixed for several periods)

- Expectations on how monetary policy will be conducted in the future are important for the inflation rate prevailing today

Central bank loss function

Assume the following loss function

$$L = \frac{1}{2}\left[(\pi - \pi^*)^2 + \omega\,(x - x^*)^2\right]$$

- $\pi^* \geq 0$ is the inflation target
- $x^* \geq 0$ is the optimal (desired) level of the output gap
- ω is the weight in the CB objective function assigned to changes in the output gap relative to inflation

Central bank loss function, cont.

- CB wish to minimize

  1. deviations of inflation from target

  1. deviations of output gap from a certain level considered optimal (for example, if you have monopolistic competition the natural level of output is lower than the efficient level of output that would arise with perfect competition and CB may wish to target the efficient level of output)

- Quadratic form:

  – Positive and negative deviations of inflation and output from targets penalized symmetrically

  – Go 2% above target is penalized the same as go 2% below target

Central bank problem

Central bank chooses the nominal interest rate *i* to minimize

$$L = \frac{1}{2}\left[(\pi - \pi^*)^2 + a(x - x^*)^2\right]$$

subject to the structure of the economy

$$(PC): \quad \pi = \pi^e + \lambda x + v$$

$$(IS): \quad x = x^e - [(i - \pi^e) - r^n] + g$$

Aside on the dynamic IS curve in terms of output gap

- Consider the dynamic IS curve

$$y = y^e - [(i - \pi^e) - \rho]$$
$$= y^e - [r - \rho]$$

- Evaluate it at the equilibrium with flexible prices

$$y^n = y^{n,e} - [r^n - \rho]$$

- Subtract the second equation from the first and add a demand shock g (for example, a shock to public spending)

$$x = x^e - [(i - \pi^e) - r^n] + g$$

Central bank problem, cont.

- Reduced problem: CB controls the output gap x and inflation π in order to minimize

$$L = \frac{1}{2}\left[(\pi - \pi^*)^2 + \omega(x - x^*)^2\right]$$

subject to

$$\pi = \pi^e + \lambda x + v$$

- Later, we can find i that minimizes L by looking at the *IS*

Central bank problem, cont.

- Assume desired output gap is zero, CB is happy with the natural output: x*=0
- Assume CB takes expectations of inflation π^e as given
  (called “discretionary optimization” as the CB chooses MP after agents have formed their expectations and acts under discretion)

Substituting *x* from the PC, the loss function becomes

$$L = \frac{1}{2}\left[(\pi - \pi^*)^2 + \omega \left(\frac{\pi - f}{\lambda} \right)^2 \right]$$

where *f* is an exogenous term from the viewpoint of the CB

$$f \equiv \pi^e + v$$

First order condition

- The FOC with respect to inflation is

$$(\pi - \pi^*) + \omega\left(\frac{\pi - f}{\lambda}\right)\frac{1}{\lambda} = 0$$

Alternatively

$$(\pi - \pi^*) + \frac{\omega}{\lambda}x = 0$$

- The FOC defines the optimal rule of monetary policy

- We call this relation btw inflation and output gap: MPR (monetary policy rule) curve

Monetary policy rule

Rewrite the FOC to obtain the MPR as

$$\pi = \pi^* - \delta x$$

where

$$\delta = \frac{\omega}{\lambda}$$

is the slope of the MPR curve

Monetary policy rule, cont.

- Implications for monetary policy conduct:
  - When $\pi > \pi^*$, CB must generate a negative output gap $x<0$
  - When $\pi < \pi^*$, CB must generate a positive output gap $x>0$
- CB should contract output below the natural level when inflation exceeds the target
- Contraction of output below the natural level reduces inflation through the supply curve
- This policy is referred to as “leaning against the wind”

Monetary policy rule, cont.

- Recall the slope of the MPR

$$\delta = \frac{\omega}{\lambda}$$

- Slope affected by two factors:

1. Relative weight of output in the loss function, ω

1. Slope of the Phillips curve, λ, measuring the cost in terms of inflation that must be beard to increase the output gap by a certain percentage

PC - MPR system

- Equations PC and MPR define a system of two equations in the variables x and π

- PC

$$\pi = \pi^e + \lambda x + v$$

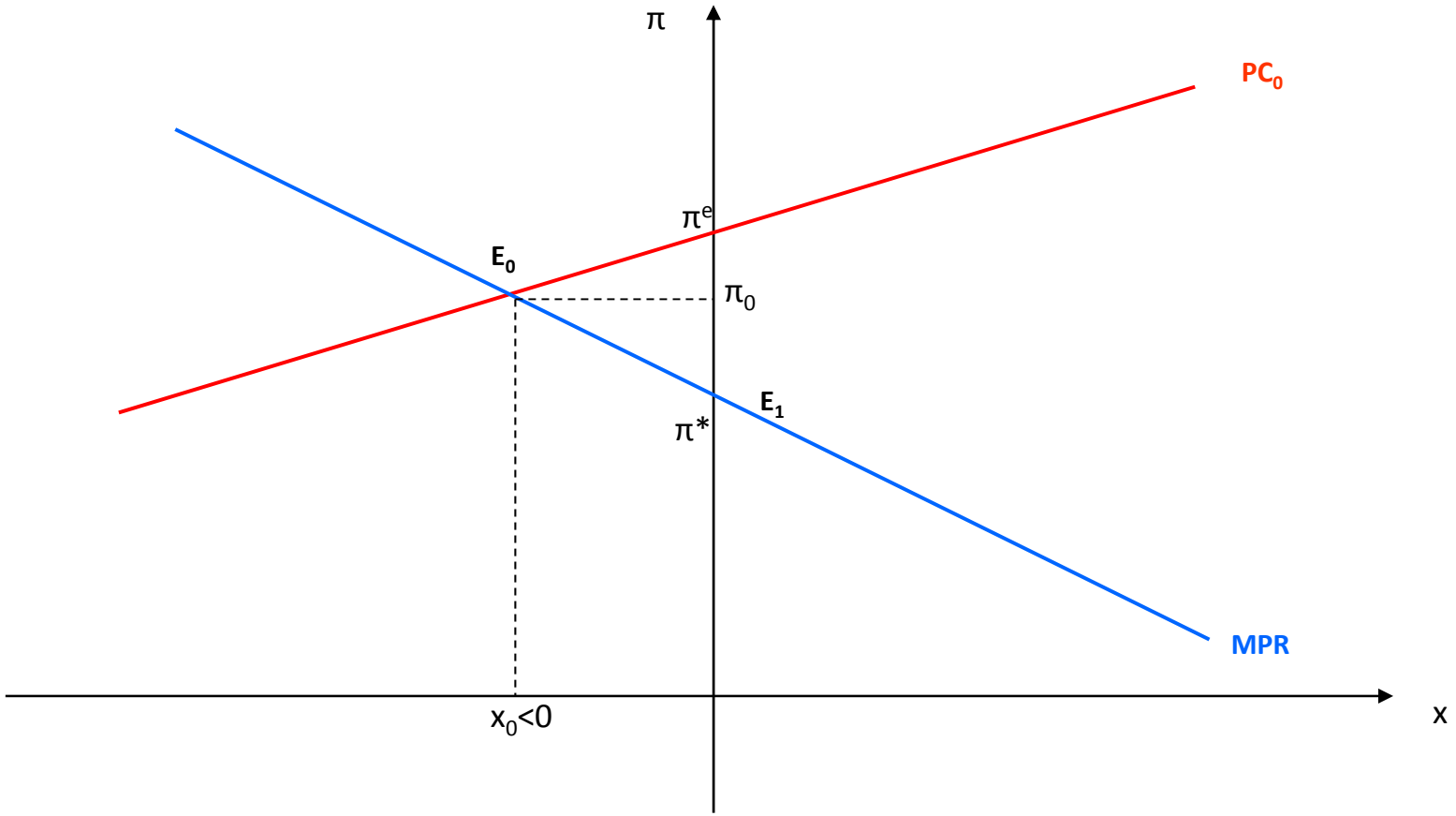
Intercept π^e; slope λ; value of the shock initially zero

- MPR

$$\pi = \pi^* - \delta x$$

Intercept π^*; slope δ

Short run equilibrium in the PC-MPR system



Equilibrium and convergence toward the target

- Assume that initially the system is in E_0

- PC and MPR cross, hence the system is in an equilibrium (x_0, π_0) satisfying both the Phillips curve PC and the monetary policy rule MPR

Short run equilibrium

Initial equilibrium corresponds to a negative output gap

$$x_0 < 0$$

and to actual inflation between target and expected inflation

$$\pi^* < \pi_0 < \pi^e$$

Short run equilibrium, cont.

- Equilibrium consistent with PC
  At output gap $x_0 < 0$ firms set prices such that inflation below expected inflation, $\pi_0 < \pi^e$

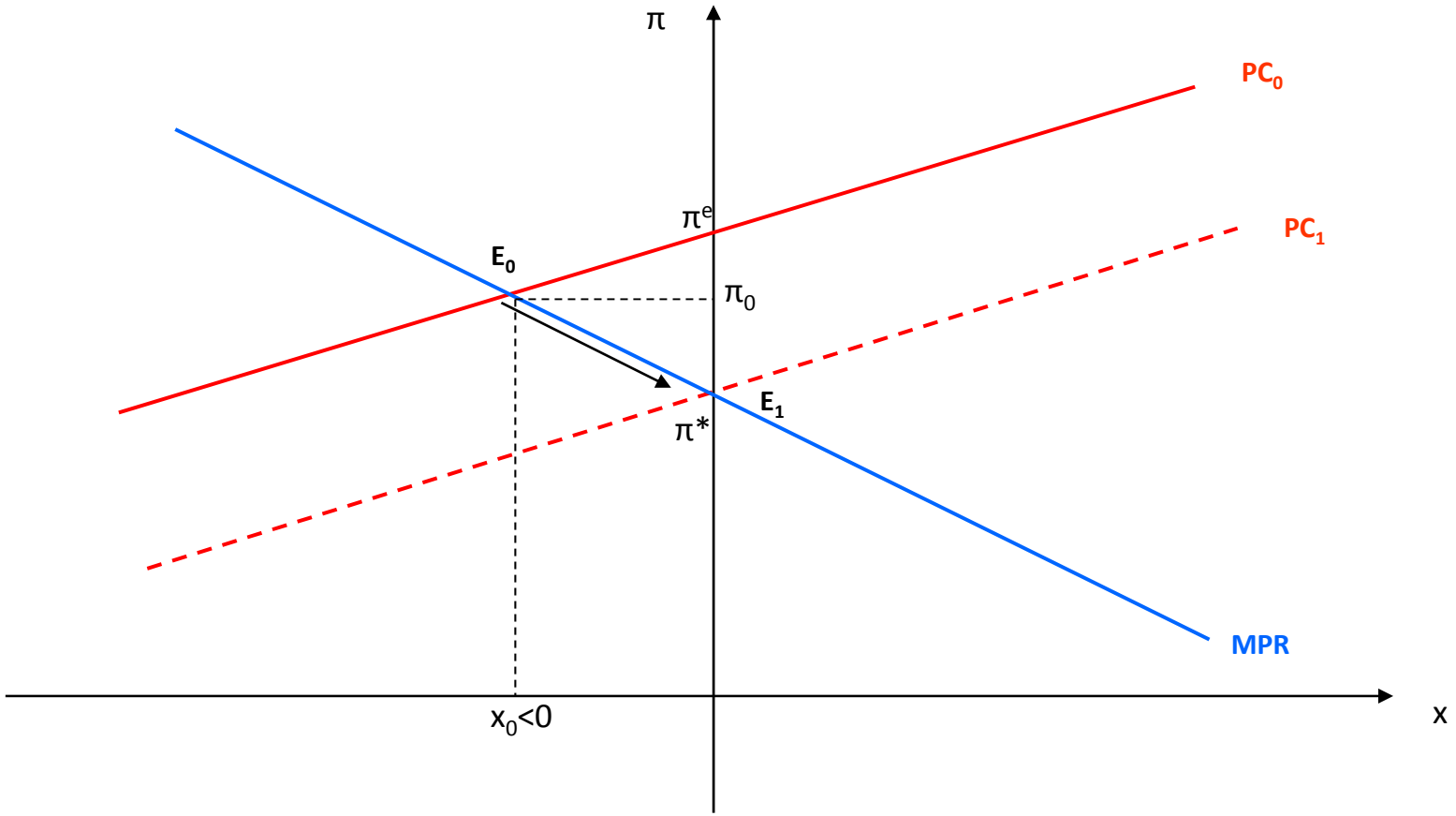
- Equilibrium consistent with MPR
  CB willing to accept output gap $x_0 < 0$ because inflation above inflation target, $\pi_0 > \pi^*$
  That is, CB is willing to accept a recession because it views inflation as too high

- Moreover, equating PC and MPR we can write

$$x_0 = \frac{\pi^* - \pi^e}{\delta + \lambda}$$

so that $\pi^e > \pi^*$ implies $x_0 < 0$

Short and long run equilibrium in the PC-MPR system



Convergence to long run equilibrium

- Point E_0 cannot be a long run equilibrium
- Over time, if agents observe that $\pi < \pi^e$ persistently, they revise their expectations
- That is, agents gradually revise their expectations downwards
- The reduction in π^e shifts downwards PC (from PC_0 to PC_1) since its intercept depends from π^e

Convergence to long run equilibrium, cont.

- The downward shifts lowers actual inflation for each value of the output gap
- With inflation lower, the central bank opts for a more expansionary policy stance
- Short run equilibrium moves toward lower inflation and higher output (smaller but still negative x)
- Convergence occurs in the long run equilibrium E_1 where actual and expected inflation coincide

$$\pi_1 = \pi^e$$

- This basically corresponds to the assumption of rational expectations: actual and expected inflation cannot diverge for long, as agents observing systematically actual inflation different from expected will revise their expectations accordingly

Long run equilibrium

- At the same time, from the PC, in E_1 we have

$$x_1 = 0$$

- Thus, from the MPR curve

$$\pi_1 = \pi^*$$

- This implies in the long run equilibrium

$$\pi_1 = \pi^e = \pi^*$$

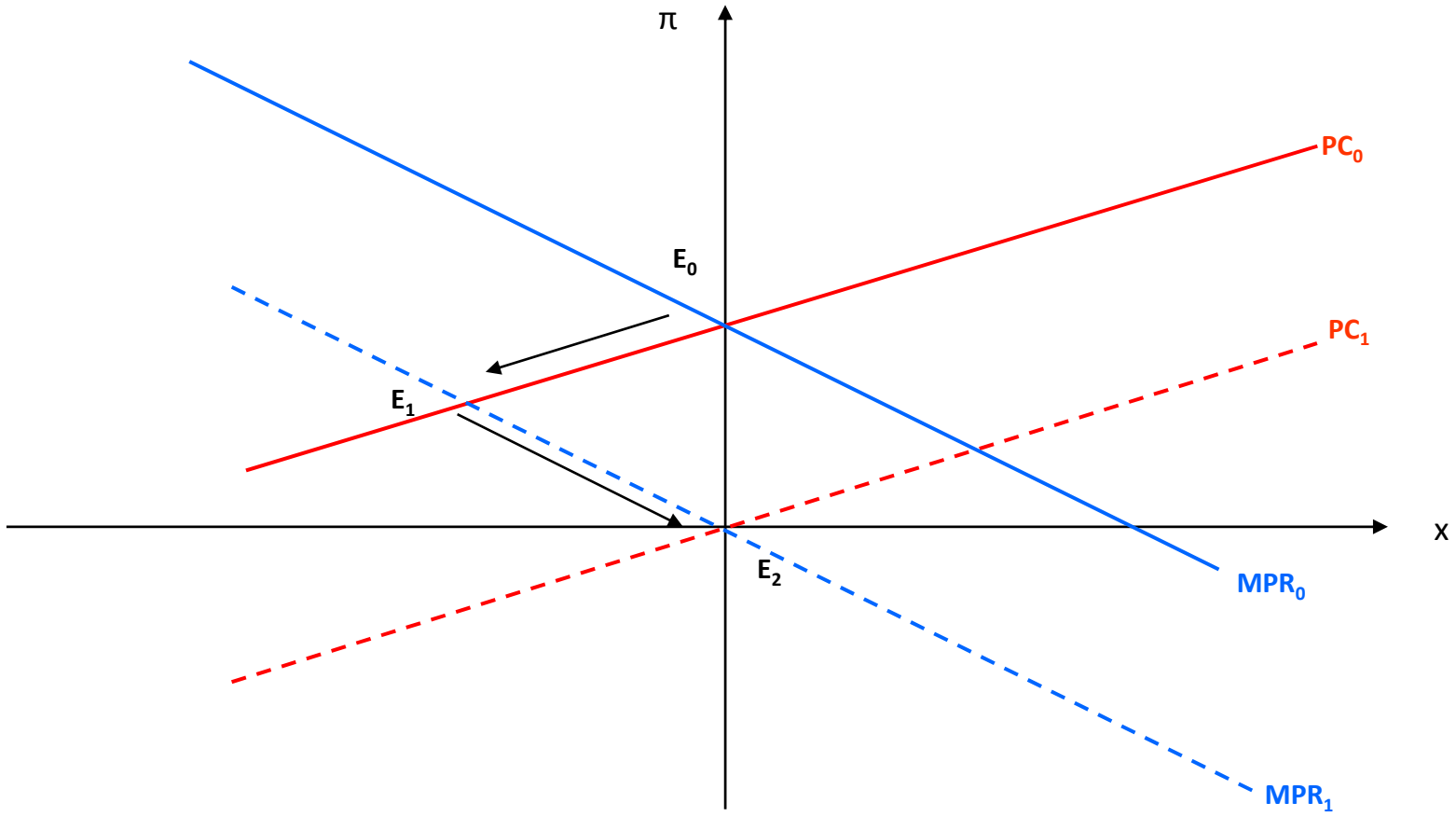
Effects of a reduction in the inflation target

- Suppose initially the economy is in the long run equilibrium E_0 with inflation target $\pi^* > 0$
- In E_0 we have

$$\pi_0 = \pi^e = \pi^*$$

- Suppose a new CB governor is nominated who decide to reduce inflation target to $\pi^{*N} = 0$

Effects of a reduction in the inflation target



Effects of a reduction in the inflation target, cont.

- Initially, the effect is a shift downward of the MPR curve, since its intercept is the inflation target
- Equilibrium E_1 corresponds to a short run equilibrium
- In such equilibrium the economy is in a recession, that is $x_1 < 0$
- Economy moves along PC curve; thus, in E_1 inflation expectations have not yet changed

Effects of a reduction in the inflation target, cont.

- Equilibrium E_1 is a short run equilibrium in which

$$\pi^{N*} < \pi_1 < \pi^e$$

- Thus expectations will move gradually downward, shifting the PC curve from its initial position PC_0 to its long run position PC_1

- Over the adjustment toward the long run equilibrium the output gap increases gradually

Effects of a reduction in the inflation target, cont.

- In the final equilibrium E_2, expectations are in line with the new target and the output gap is again zero:

$$\pi^{N*} = \pi_2 = \pi^e$$

$$x_2 = 0$$

- Note that CB credibility can influence on the convergence process

- The more credible is the inflation target $\pi^{N*} = 0$ the more rapid will the convergence be

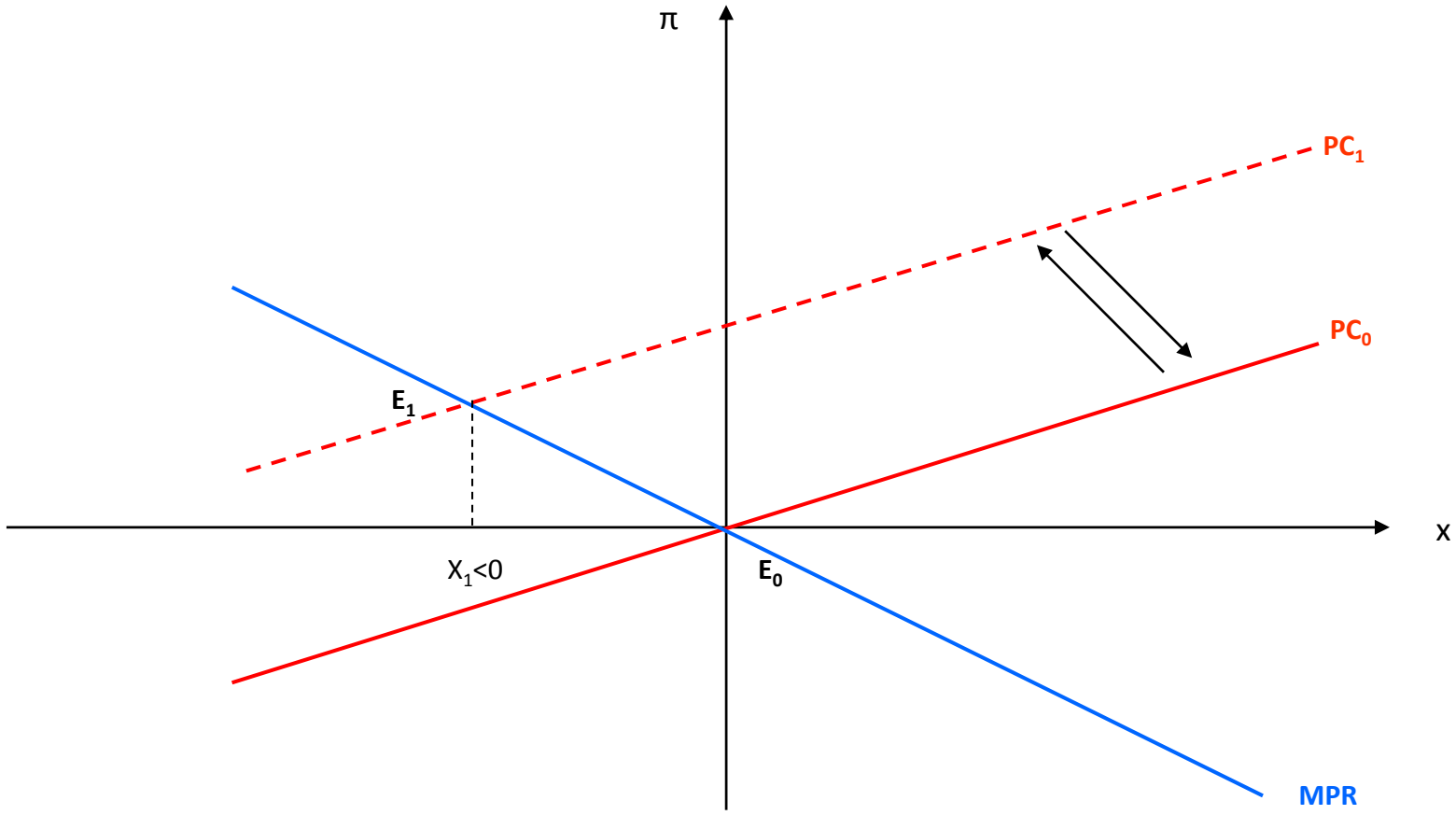
Shock to expected inflation

- Suddenly agents expect that inflation will increase in the future (even though target unchanged)
- Initially the economy is in the long run equilibrium E_0 with

$$\pi_0 = \pi^e = \pi^* = 0$$

- An increase in expected inflation shifts the PC curve upward
- The short run equilibrium is in E_1 with $\pi_1 > \pi^*$ and $x_1 < 0$

Effects of an increase in expected inflation



Shock to expected inflation, cont.

- Long run dynamics once more guided by the relation between target and expected inflation
- In E_1, in fact, we have

$$\pi^* < \pi_1 < \pi^e$$

- Expected inflation cannot diverge for long from target inflation
- Thus expected inflation starts moving down shifting at the same time the PC curve gradually toward its initial position

Shock to expected inflation, cont.

- To sum up, an increase in expected inflation determines, with inflation targeting, a temporary phase of negative output gap

- Note that CB credibility can influence on the convergence process

- The more credible is the inflation target $\pi^* = 0$ the more rapid will the convergence be

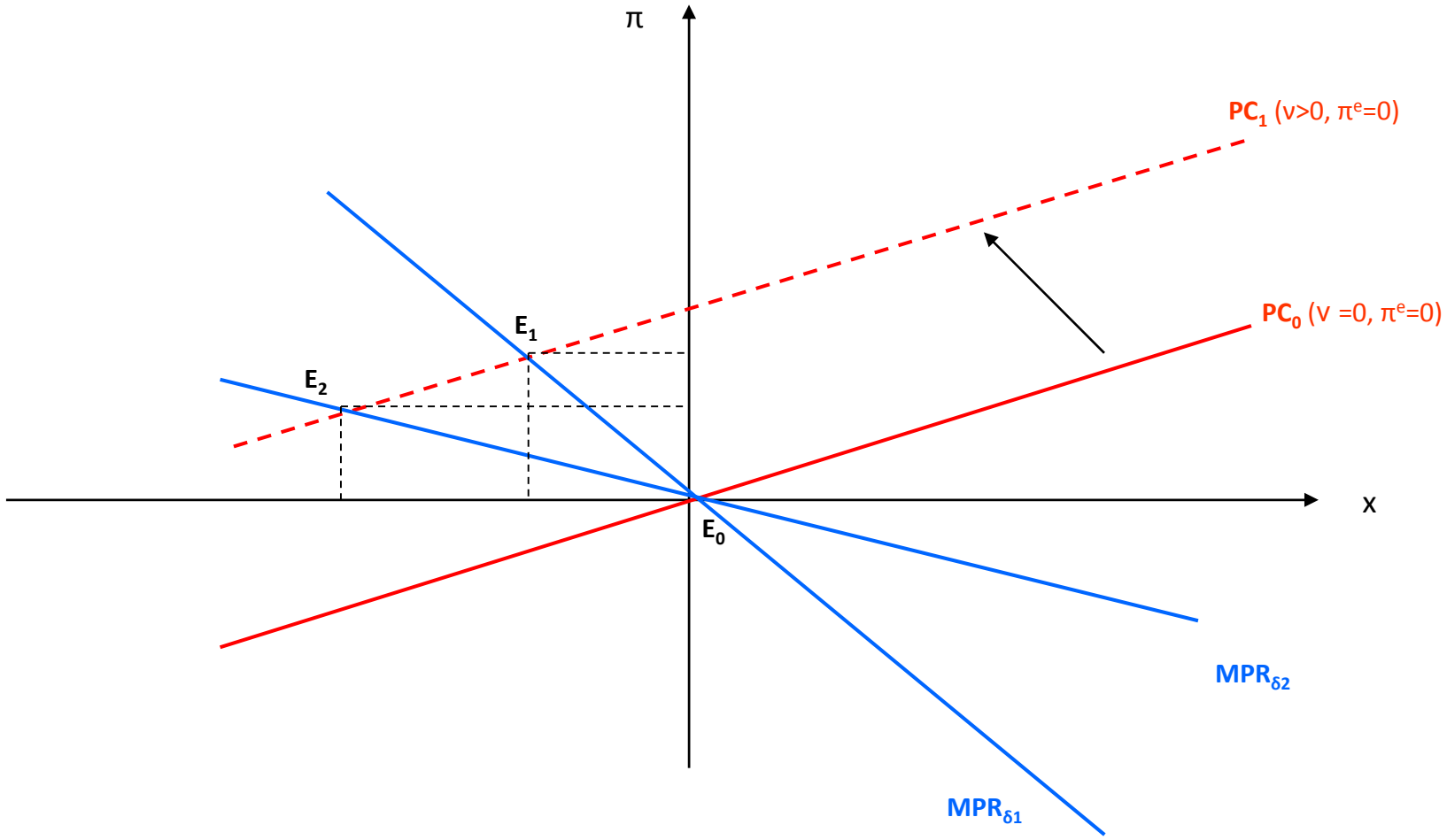
Preferences of CB and supply shock

- Represent two different curves $MPR_{\delta 1}$ and $MPR_{\delta 2}$ corresponding respectively to a slope $\delta_1 > \delta_2$

- The curve with the higher slope $MPR_{\delta 1}$ corresponds to the case of a CB placing larger weight on output gap stabilization relative to inflation stabilization than the curve with the lower slope $MPR_{\delta 2}$

- $\delta_1 > \delta_2 \rightarrow \omega_1 > \omega_2$
  $MPR_{\delta 1} \rightarrow$ relatively less conservative CB
  $MPR_{\delta 2} \rightarrow$ relatively more conservative CB

Preferences of CB and supply shock



Preferences of CB and supply shock, cont.

- Consider now the case of a temporary positive supply shock $v > 0$
- The shock hits only in the current period, then it returns to its zero value
- Such a shock moves the PC curve up from PC_0 to PC_1: at each value of the output gap corresponds now a higher value of inflation
- In other words, inflation increases for reasons that are independent from the output gap and expected inflation

Preferences of CB and supply shock, cont.

- Note that expected inflation does not change and remains at zero because the shock is temporary: since the shock only hits in the current period and agents know it will go back to zero in the future, there is no reason to change expected inflation

- The CB responds to the shock: it tries to limit the inflationary pressures through a reduction in the output gap

- In presence of a supply shock the CB is unable to achieve the optimal combination of inflation and output gap, given by $\pi = \pi^*$ and $x = x^* = 0$

- This happens even though expected inflation equals target inflation

Preferences of CB and supply shock, cont.

- CB faces a trade-off between how much increase of inflation above target and how much reduction of output below the natural level it wants to tolerate

- By accepting a large fall in the output gap, the CB can limit the increase in inflation; by accepting a large increase in inflation, the CB can limit the fall of the output gap

Preferences of CB and supply shock, cont.

Depending on the CB preferences for output stabilization relative to inflation stabilization (as represented by the weight ω) the supply shock will either cause

- a large increase in inflation and a small fall in the output gap $\Rightarrow$ this occurs for the less conservative CB (δ_1 high)

- a small increase in inflation and a large fall in the output gap $\Rightarrow$ this occurs for the more conservative CB (δ_2 low)