STATISTICAL METHODS

Master in Industrial Management,

Operations and Sustainability (MIMOS)

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https://doity.com.br/estatistica-aplicada-a-nutricao


https://basiccode.com.br/produto/informatica-basica/

PROGRAM

1. Fundamental Concepts of Statistics
2. Descriptive Data Analysis
3. Introduction to Inferential Analysis
4. Parametric Hypothesis Testing
5. Non-Parametric Hypothesis Testing
6. Linear Regression Analysis

HOMEWORK OF LECTURE 3: QUESTIONS

EXERCISE 3.17

3.17 A department store manager has monitored the number of complaints received per week about poor service. The probabilities for numbers of complaints in a week, established by this review, are shown in the following table. Let *A* be the event "there will be at least one complaint in a week" and *B* the event "there will be fewer than ten complaints in a week."

Number of complaints	0	1 to 3	4 to 6	7 to 9	10 to 12	More than 12
Probability	0.14	0.39	0.23	0.15	0.06	0.03

a. Find the probability of *A*.
b. Find the probability of *B*.
c. Find the probability of the complement of *A*.
d. Find the probability of the union of *A* and *B*.
e. Find the probability of the intersection of *A* and *B*.
f. Are *A* and *B* mutually exclusive?
g. Are *A* and *B* collectively exhaustive?

Newbold et al (2013)

EXERCISE 3.17 A): SOLUTION

✅ Answer:

Given table:

Number of complaints	0	1–3	4–6	7–9	10–12	More than 12
Probability	0.14	0.39	0.23	0.15	0.06	0.03

Let:

- $A =$ "there will be **at least one complaint** in a week"
- $B =$ "there will be **fewer than ten complaints** in a week"

a. Probability of A

Event A = at least one complaint = all outcomes **except 0 complaints**.

$$P(A) = 1 - P(0) = 1 - 0.14 = 0.86$$

EXERCISE 3.17 B): SOLUTION

Answer:

Given table:

Number of complaints	0	1–3	4–6	7–9	10–12	More than 12
Probability	0.14	0.39	0.23	0.15	0.06	0.03

Let:

- $A =$ "there will be **at least one complaint** in a week"
- $B =$ "there will be **fewer than ten complaints** in a week"

b. Probability of B

Event B = fewer than 10 complaints = sum of probabilities for 0, 1–3, 4–6, 7–9 complaints:

$$P(B) = 0.14 + 0.39 + 0.23 + 0.15 = 0.91$$

EXERCISE 3.17 C): SOLUTION

✅ Answer:

Given table:

Number of complaints	**0**	**1–3**	**4–6**	**7–9**	**10–12**	**More than 12**
Probability	0.14	0.39	0.23	0.15	0.06	0.03

Let:

- $A =$ "there will be **at least one complaint** in a week"
- $B =$ "there will be **fewer than ten complaints** in a week"

c. Probability of the complement of A

Complement of A = "no complaints in a week" = 0 complaints:

$$P(A^c) = P(0) = 0.14$$

EXERCISE 3.17 D): SOLUTION

✅ Answer:

Given table:

Number of complaints	**0**	**1–3**	**4–6**	**7–9**	**10–12**	**More than 12**
Probability	0.14	0.39	0.23	0.15	0.06	0.03

Let:

- $A =$ "there will be **at least one complaint** in a week"
- $B =$ "there will be **fewer than ten complaints** in a week"

d. Probability of the union of A and B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

We still need $P(A \cap B)$.

EXERCISE 3.17 E): SOLUTION

Answer:

Given table:

Number of complaints	**0**	**1–3**	**4–6**	**7–9**	**10–12**	**More than 12**
Probability	0.14	0.39	0.23	0.15	0.06	0.03

Let:

- $A =$ "there will be **at least one complaint** in a week"
- $B =$ "there will be **fewer than ten complaints** in a week"

e. Probability of the intersection of A and B

Intersection $A \cap B$ = "at least one complaint **and** fewer than 10 complaints" = outcomes 1–3, 4–6, 7–9:

$$P(A \cap B) = 0.39 + 0.23 + 0.15 = 0.77$$

Now we can compute the union:

$$P(A \cup B) = 0.86 + 0.91 - 0.77 = 1.00$$

EXERCISE 3.17 F): SOLUTION

✅ Answer:

Given table:

Number of complaints	**0**	**1–3**	**4–6**	**7–9**	**10–12**	**More than 12**
Probability	0.14	0.39	0.23	0.15	0.06	0.03

Let:

- $A =$ "there will be **at least one complaint** in a week"
- $B =$ "there will be **fewer than ten complaints** in a week"

f. Are A and B mutually exclusive?

- Two events are **mutually exclusive** if they **cannot occur together**.
- $\boldsymbol{P(A \cap B) = 0.77 \neq 0}$, so **not mutually exclusive**.

EXERCISE 3.17 G): SOLUTION

✅ Answer:

Given table:

Number of complaints	**0**	**1–3**	**4–6**	**7–9**	**10–12**	**More than 12**
Probability	0.14	0.39	0.23	0.15	0.06	0.03

Let:

- $A =$ "there will be **at least one complaint** in a week"
- $B =$ "there will be **fewer than ten complaints** in a week"

g. Are A and B collectively exhaustive?

- Two events are **collectively exhaustive** if their **union covers all outcomes**.
- $P(A \cup B) = 1$, so **yes, they are collectively exhaustive**.

EXERCISE 3.13

3.13 A corporation has just received new machinery that must be installed and checked before it becomes operational. The accompanying table shows a manager's probability assessment for the number of days required before the machinery becomes operational.

Number of days	3	4	5	6	7
Probability	0.08	0.24	0.41	0.20	0.07

Let *A* be the event "it will be more than four days before the machinery becomes operational," and let *B* be the event "it will be less than six days before the machinery becomes available."

a. Find the probability of event *A*.
b. Find the probability of event *B*.
c. Find the probability of the complement of event *A*.
d. Find the probability of the intersection of events *A* and *B*.
e. Find the probability of the union of events *A* and *B*.

Newbold et al (2013)

EXERCISE 3.13 A): SOLUTION

✅ Answer:

Given table:

Number of days	3	4	5	6	7
Probability	0.08	0.24	0.41	0.20	0.07

Let:

- $A =$ "more than 4 days before the machinery becomes operational" → days 5, 6, 7
- $B =$ "less than 6 days before the machinery becomes available" → days 3, 4, 5

a. Probability of event A

$$P(A) = P(5) + P(6) + P(7) = 0.41 + 0.20 + 0.07 = 0.68$$

EXERCISE 3.13 B): SOLUTION

✅ Answer:

Given table:

Number of days	**3**	**4**	**5**	**6**	**7**
Probability	0.08	0.24	0.41	0.20	0.07

Let:

- $A =$ "more than 4 days before the machinery becomes operational" → days 5, 6, 7
- $B =$ "less than 6 days before the machinery becomes available" → days 3, 4, 5

b. Probability of event B

$$P(B) = P(3) + P(4) + P(5) = 0.08 + 0.24 + 0.41 = 0.73$$

EXERCISE 3.13 C): SOLUTION

✅ Answer:

Given table:

Number of days	**3**	**4**	**5**	**6**	**7**
Probability	0.08	0.24	0.41	0.20	0.07

Let:

- $A =$ "more than 4 days before the machinery becomes operational" → days 5, 6, 7
- $B =$ "less than 6 days before the machinery becomes available" → days 3, 4, 5

c. Probability of the complement of event A

Complement of A = "4 days or less" → days 3, 4:

$$P(A^c) = P(3) + P(4) = 0.08 + 0.24 = 0.32$$

EXERCISE 3.13 D): SOLUTION

✅ Answer:

Given table:

Number of days	**3**	**4**	**5**	**6**	**7**
Probability	0.08	0.24	0.41	0.20	0.07

Let:

- $A =$ "more than 4 days before the machinery becomes operational" → days 5, 6, 7
- $B =$ "less than 6 days before the machinery becomes available" → days 3, 4, 5

d. Probability of the intersection of events A and B

Intersection $A \cap B$ = "more than 4 days **and** less than 6 days" → only day 5:

$$P(A \cap B) = P(5) = 0.41$$

EXERCISE 3.13 E): SOLUTION

Answer:

Given table:

Number of days	**3**	**4**	**5**	**6**	**7**
Probability	0.08	0.24	0.41	0.20	0.07

Let:

- $A =$ "more than 4 days before the machinery becomes operational" → days 5, 6, 7
- $B =$ "less than 6 days before the machinery becomes available" → days 3, 4, 5

e. Probability of the union of events A and B

Use the **addition rule**:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.68 + 0.73 - 0.41 = 1.00$$

EXERCISE 3.14

3.14 On a sample of 1,500 people in Sydney, Australia, 89 have no credit cards (event A), 750 have one (event B), 450 have two (event C) and the rest have more than two (event D). On the basis of the data, calculate each of the following.

a. The probability of event A
b. The probability of event D
c. The complement of event B
d. The complement of event C
e. The probability of event A or D

Newbold et al (2013)

EXERCISE 3.24 A): SOLUTION

Answer:

We are given a sample of **1,500 people** in Sydney:

- Event A: 89 people have no credit cards
- Event B: 750 people have one credit card
- Event C: 450 people have two credit cards
- Event D: the rest have more than two

Find the number of people in event D

$$\text{Number in D} = 1500 - (89 + 750 + 450) = 1500 - 1289 = 211$$

a. Probability of event A

$$P(A) = \frac{\text{number in A}}{\text{total}} = \frac{89}{1500} \approx 0.0593$$

EXERCISE 3.24 B): SOLUTION

Answer:

We are given a sample of **1,500 people** in Sydney:

- Event A: 89 people have no credit cards
- Event B: 750 people have one credit card
- Event C: 450 people have two credit cards
- Event D: the rest have more than two

Find the number of people in event D

$$\text{Number in D} = 1500 - (89 + 750 + 450) = 1500 - 1289 = 211$$

b. Probability of event D

$$P(D) = \frac{211}{1500} \approx 0.1407$$

EXERCISE 3.24 C): SOLUTION

✅ Answer:

We are given a sample of **1,500 people** in Sydney:

- Event A: 89 people have no credit cards
- Event B: 750 people have one credit card
- Event C: 450 people have two credit cards
- Event D: the rest have more than two

Find the number of people in event D

$$\text{Number in D} = 1500 - (89 + 750 + 450) = 1500 - 1289 = 211$$

c. Complement of event B

$$P(B^c) = 1 - P(B) = 1 - \frac{750}{1500} = 1 - 0.5 = 0.5$$

EXERCISE 3.24 D): SOLUTION

Answer:

We are given a sample of **1,500 people** in Sydney:

- Event A: 89 people have no credit cards
- Event B: 750 people have one credit card
- Event C: 450 people have two credit cards
- Event D: the rest have more than two

Find the number of people in event D

$$\text{Number in D} = 1500 - (89 + 750 + 450) = 1500 - 1289 = 211$$

d. Complement of event C

$$P(C^c) = 1 - P(C) = 1 - \frac{450}{1500} = 1 - 0.3 = 0.7$$

EXERCISE 3.24 E): SOLUTION

✅ Answer:

We are given a sample of **1,500 people** in Sydney:

- Event A: 89 people have no credit cards
- Event B: 750 people have one credit card
- Event C: 450 people have two credit cards
- Event D: the rest have more than two

Find the number of people in event D

$$\text{Number in D} = 1500 - (89 + 750 + 450) = 1500 - 1289 = 211$$

e. Probability of event A or D

Since A and D are mutually exclusive:

$$P(A \text{ or } D) = P(A) + P(D) = 0.0593 + 0.1407 = 0.20$$

EXERCISE 3.70

3.70 The accompanying table shows proportions of adults in metropolitan areas, categorized as to whether they are public-radio contributors and whether or not they voted in the last election.

Voted	Contributors	Noncontributors
Yes	0.63	0.13
No	0.14	0.10

a. What is the probability that a randomly chosen adult from this population voted?
b. What is the probability that a randomly chosen adult from this population contributes to public radio?
c. What is the probability that a randomly chosen adult from this population did not contribute and did not vote?

Newbold et al (2013)

EXERCISE 3.70 A): SOLUTION

Answer:

Given table:

Voted	Contributors	Noncontributors
Yes	0.63	0.13
No	0.14	0.10

a. Probability that a randomly chosen adult voted

$$P(\text{Voted}) = P(\text{Yes, Contributor}) + P(\text{Yes, Noncontributor}) = 0.63 + 0.13 = 0.76$$

Answer (a): 0.76

EXERCISE 3.70 B): SOLUTION

✅ Answer:

Given table:

Voted	Contributors	Noncontributors
Yes	0.63	0.13
No	0.14	0.10

b. Probability that a randomly chosen adult contributes to public radio

$$P(\text{Contributor}) = P(\text{Yes, Contributor}) + P(\text{No, Contributor}) = 0.63 + 0.14 = 0.77$$

✅ **Answer (b):** 0.77

EXERCISE 3.70 C): SOLUTION

Answer:

Given table:

Voted	Contributors	Noncontributors
Yes	0.63	0.13
No	0.14	0.10

c. Probability that a randomly chosen adult did not contribute and did not vote

$$P(\text{Noncontributor and No}) = 0.10$$

Answer (c): 0.10

LECTURE 4: PROBABILITY (CONTINUED)

CONDITIONAL PROBABILITY

- A conditional probability is the probability of one event, given that another event has occurred:

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}} \Rightarrow$$

The conditional probability of *A* given that *B* has occurred

$$\boxed{P(B|A) = \frac{P(A \cap B)}{P(A)}} \Rightarrow$$

The conditional probability of *B* given that *A* has occurred

Newbold et al (2013)

Complementary Probabilities

$P(\bar{A}|B) = 1 - P(A|B)$

$P(\bar{B}|A) = 1 - P(B|A)$

CONDITIONAL PROBABILITY EXAMPLE

- **Of the cars on a used car lot, 70% have air conditioning (AC) and 40% have a CD player (CD). 20% of the cars have both.**
- What is the probability that a car has a CD player, given that it has AC?

Newbold et al (2013)

CONDITIONAL PROBABILITY EXAMPLE: SOLUTION

- Of the cars on a used car lot, 70% have air conditioning (AC) and 40% have a CD player (CD). 20% of the cars have both.

	CD	No CD	Total
AC	.2	.5	.7
No AC	.2	.1	.3
Total	.4	.6	1.0

$P(AC) = 0.7$
$P(CD) = 0.4$
$P(CD \cap AC) = 0.2$

Newbold et al (2013)

MULTIPLICATION RULE

- Multiplication rule for two events *A* and *B*:

$$P(A \cap B) = P(A \mid B)P(B)$$

- also

$$P(A \cap B) = P(B \mid A)P(A)$$

Conditional Probabilities

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Newbold et al (2013)

The probability of the intersection of two events A and B can be expressed as:

$$P(A \cap B) = P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$$

- $P(A|B)$: Conditional probability of A given B
- $P(B|A)$: Conditional probability of B given A
- $P(A \cap B)$: Joint probability of A and B

STATISTICAL INDEPENDENCE: TWO EVENTS

- If A and B are independent, then

$$P(A \mid B) = P(A) \quad \textbf{if } P(B) > 0$$
$$P(B \mid A) = P(B) \quad \textbf{if } P(A) > 0$$

Events A and B **are independent** when the probability of one event is not affected by the other event.

- Two events are statistically independent if and only if:

$$P(A \cap B) = P(A)P(B)$$

Newbold et al (2013)

This is the general formula for the joint probability (or probability of intersection) of two events.

$$P(A \cap B) = P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$$

STATISTICAL INDEPENDENCE: MULTIPLE EVENTS

- For multiple events:

$E_1, E_2, \ldots, E_k$ are statistically independent if and only if:

$$\boxed{P(E_1 \cap E_1 \cap \ldots \cap E_1) = P(E_1)P(E_2)\ldots P(E_k)}$$

Newbold et al (2013)

STATISTICAL INDEPENDENCE EXAMPLE

- Of the cars on a used car lot, 70% have air conditioning (AC) and 40% have a CD player (CD). 20% of the cars have both.

	CD	No CD	Total
AC	.2	.5	.7
No AC	.2	.1	.3
Total	.4	.6	1.0

- Are the events AC and CD statistically independent?

Newbold et al (2013)

STATISTICAL INDEPENDENCE EXAMPLE: SOLUTION

	CD	No CD	Total
AC	.2	.5	.7
No AC	.2	.1	.3
Total	.4	.6	1.0

$$P(AC \cap CD) = 0.2$$

$$\left.\begin{matrix} P(AC)=0.7 \\ P(CD)=0.4 \end{matrix}\right\} P(AC)P(CD) = (0.7)(0.4) = 0.28$$

$$P(AC \cap CD) = 0.2 \neq P(AC)P(CD) = 0.28$$

So the two events are not statistically independent

Newbold et al (2013)

EXERCISE 3.27

3.27 A company knows that a rival is about to bring out a competing product. It believes that this rival has three possible packaging plans (superior, normal, and cheap) in mind and that all are equally likely. Also, there are three equally likely possible marketing strategies (intense media advertising, price discounts, and the use of a coupon to reduce the price of future purchases). What is the probability that the rival will employ superior packaging in conjunction with an intense media advertising campaign? Assume that packaging plans and marketing strategies are determined independently.

Newbold et al (2013)

EXERCISE 3.27: SOLUTION

✅ Answer:

Given:

- **Packaging plans:** Superior, Normal, Cheap → all equally likely.

$$P(\text{Superior}) = \frac{1}{3}$$

- **Marketing strategies:** Intense media advertising, Price discounts, Coupon → all equally likely.

$$P(\text{Intense media advertising}) = \frac{1}{3}$$

- **Independence:** Packaging and marketing decisions are independent.

Step 1: Identify the event

We want the probability of:

$$\text{Superior packaging AND Intense media advertising}$$

Step 2: Use the multiplication rule for independent events

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$P(\text{Superior AND Intense}) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

PARTITION OF A SET

A *partition* of a sample space S is a collection of subsets

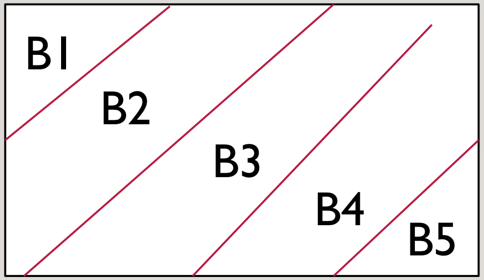
$$B_1, B_2, \ldots, B_n$$

such that:

1. $B_i \cap B_j = \varnothing$ for all $i \neq j$ (they are **mutually exclusive**),
2. $B_1 \cup B_2 \cup \cdots \cup B_n = S$ (they are **collectively exhaustive**).

A partition divides a set into non-overlapping pieces that together include every element of the set.

Example: Suppose we have a partition of the sample space into 5 sets $B_1, B_2, \ldots, B_5$.



Sample Space S

THE LAW OF TOTAL PROBABILITY

"Let S be a partition of the sample space into sets $B_1, B_2, \ldots, B_n$, which are mutually exclusive and collectively exhaustive. Then, for any event A, we have:"

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + \cdots + P(A \cap B_n)$$

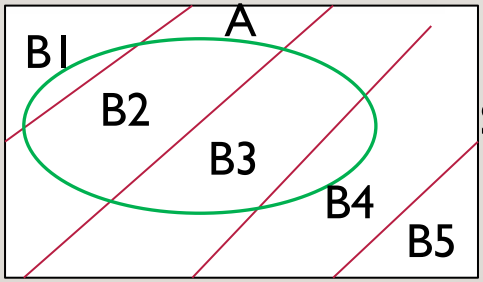
$$= P(A \mid B_1)\, P(B_1) + P(A \mid B_2)\, P(B_2) + \cdots + P(A \mid B_n)\, P(B_n).$$

$$P(A) = \sum_{i=1}^{n} P(A \mid B_i)\, P(B_i)$$

Example: Suppose we have a partition of the sample space into 5 sets $B_1, B_2, \ldots, B_5$.
If we want to find the probability of an event A, we apply the Law of Total Probability:

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3) + P(A \cap B_4) + P(A \cap B_5)$$

$$= P(A \mid B_1)\, P(B_1) + P(A \mid B_2)\, P(B_2) + P(A \mid B_3)\, P(B_3) + P(A \mid B_4)\, P(B_4) + P(A \mid B_5)\, P(B_5).$$



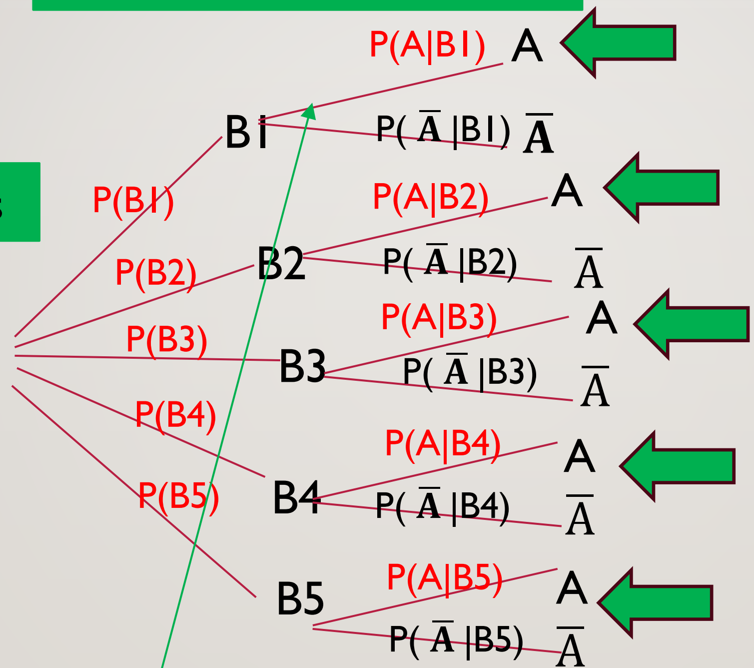
Sample Space S

TREE DIAGRAM: THE LAW OF TOTAL PROBABILITY

Conditional Probabilities

Joint Probabilities

Marginal Probabilities



$$P(A \cap B_1) = P(A|B_1) \cdot P(B_1)$$

$$P(A \cap B_2) = P(A|B_2) \cdot P(B_2)$$

$$P(A \cap B_3) = P(A|B_3) \cdot P(B_3)$$

$$P(A \cap B_4) = P(A|B_4) \cdot P(B_4)$$

$$P(A \cap B_5) = P(A|B_5) \cdot P(B_5)$$

Note: The probabilities must sum to 1 because the branches represent all mutually exclusive outcomes from that node, and one of them must occur.

If I have a partition $B_1, \ldots, B_5$ and I want to find the probability of A, I only consider the paths in the tree diagram that lead to A, which is essentially an application of the **Law of Total Probability.**

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3) + P(A \cap B_4) + P(A \cap B_5)$$

$$= P(A \mid B_1)\, P(B_1) + P(A \mid B_2)\, P(B_2) + P(A \mid B_3)\, P(B_3) + P(A \mid B_4)\, P(B_4) + P(A \mid B_5)\, P(B_5).$$

JOINT AND MARGINAL PROBABILITIES: REVIEW

- The probability of a joint event, $A \cap B$:

$$P(A \cap B) = \frac{\text{number of outcomes satisfying } A \text{ and } B}{\text{total number of elementary outcomes}}$$

- Computing a marginal probability:

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + \cdots + P(A \cap B_k)$$

  - Where $B_1, B_2, \ldots, B_k$ are *k* mutually exclusive and collectively exhaustive events

Newbold et al (2013)

BAYES' THEOREM

Let A_1 and B_1 be two events. Bayes' theorem states that

$$P(B_1 \mid A_1) = \frac{P(A_1 \mid B_1)P(B_1)}{P(A_1)}$$

and

$$P(A_1 \mid B_1) = \frac{P(B_1 \mid A_1)P(A_1)}{P(B_1)}$$

- a way of revising conditional probabilities by using available or additional information

In **Bayes' theorem**, the denominator is calculated using the **Law of Total Probability**.

The Law of Total Probability

$$P(A) = \sum_{i=1}^{n} P(A \mid B_i)\, P(B_i)$$

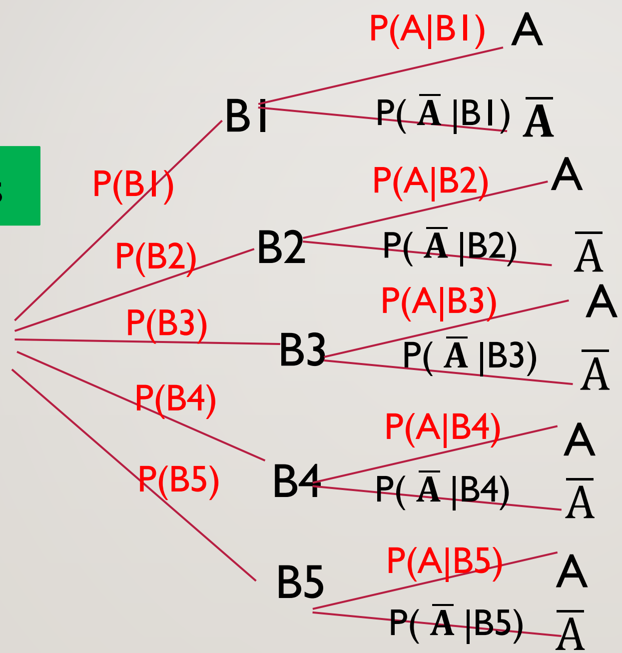
$$P(B) = \sum_{i=1}^{n} P(B \mid A_i) \cdot P(A_i)$$

Newbold et al (2013)

TREE DIAGRAM: BAYES´THEOREM

Marginal Probabilities

Conditional Probabilities



Joint Probabilities

$$P(A \cap B_1) = P(A|B_1) \cdot P(B_1)$$

$$P(A \cap B_2) = P(A|B_2) \cdot P(B_2)$$

$$P(A \cap B_3) = P(A|B_3) \cdot P(B_3)$$

$$P(A \cap B_4) = P(A|B_4) \cdot P(B_4)$$

$$P(A \cap B_5) = P(A|B_5) \cdot P(B_5)$$

Bayes’ Probabilities

$P(B_1 \mid A)$

$P(B_2 \mid A)$

$P(B_3 \mid A)$

$P(B_4 \mid A)$

$P(B_5 \mid A)$

CONDITIONAL PROBABILITY VS BAYES' PROBABILITY

"The difference between conditional probabilities and Bayes' probabilities is as follows:

- Both use the same notation $P(A|B)$.
- **Conditional probability** expresses the probability of a **future or current event given a past event**: $P(\text{Present}|\text{Past})$.
- **Bayes' probability** expresses the probability of a **past event given a present observation**: $P(\text{Past}|\text{Present})$."

Example: Consider a disease and its symptoms.

- **Past:** Disease
- **Present:** Symptoms

Type	Notation	Meaning
Conditional Probability	$P(\text{Symptoms} \mid \text{Disease})$	Probability of observing the **present** (symptoms) given the **past** (disease)
Bayes' Probability	$P(\text{Disease} \mid \text{Symptoms})$	Probability of the **past** (disease) given the **present** (observed symptoms)

- **Conditional probability**: "What is the likelihood of symptoms if the disease is present?"
- **Bayes' probability**: "What is the likelihood of the disease given the observed symptoms?"

BAYES' THEOREM EXAMPLE

The members of a consulting firm rent cars from three rental agencies: 60 percent from agency 1, 30 percent from agency 2, and 10 percent from agency 3. If 9 percent of the cars from agency 1 need an oil change, 20 percent of the cars from agency 2 need an oil change, and 6 percent of the cars from agency 3 need an oil change, what is the probability that a rental car delivered to the firm will need an oil change?

Solution

If A is the event that the car needs an oil change, and B_1, B_2, and B_3 are the events that the car comes from rental agencies 1, 2, or 3, we have $P(B_1) = 0.60$, $P(B_2) = 0.30$, $P(B_3) = 0.10$, $P(A|B_1) = 0.09$, $P(A|B_2) = 0.20$, and $P(A|B_3) = 0.06$. Substituting these values into the formula of Theorem 12, we get

The Law of Total Probability

$$P(A) = \sum_{i=1}^{n} P(A \mid B_i)\, P(B_i)$$

$$\begin{aligned} P(A) &= (0.60)(0.09) + (0.30)(0.20) + (0.10)(0.06) \\ &= 0.12 \end{aligned}$$

Thus, 12 percent of all the rental cars delivered to this firm will need an oil change.

Newbold et al (2013)

BAYES' THEOREM EXAMPLE

- If the car needed an oil change, what is the probability that it came from rental agency 2?

Solution

Substituting the probabilities on the previous page into the formula of Theorem 13, we get

$$P(B_2|A) = \frac{(0.30)(0.20)}{(0.60)(0.09) + (0.30)(0.20) + (0.10)(0.06)}$$

$$= \frac{0.060}{0.120}$$

$$= 0.5$$

Bayes´ Theorem

$$P(B_2|A) = \frac{P(A|B_2) \cdot P(B_2)}{P(A)}$$

Observe that although only 30 percent of the cars delivered to the firm come from agency 2, 50 percent of those requiring an oil change come from that agency.

Newbold et al (2013)

EXERCISE 3.83

3.83 A publisher sends advertising materials for an accounting text to 80% of all professors teaching the appropriate accounting course. Thirty percent of the professors who received this material adopted the book, as did 10% of the professors who did not receive the material. What is the probability that a professor who adopts the book has received the advertising material?

Newbold et al (2013)

EXERCISE 3.83: SOLUTION

Answer:

Given:

- $P(M) = 0.80$: professor receives advertising material
- $P(M^c) = 0.20$: professor does not receive material
- $P(A \mid M) = 0.30$: adoption if received material
- $P(A \mid M^c) = 0.10$: adoption if no material

We want:

$$P(M \mid A) = ?$$

Step 1: Apply Bayes' Theorem

Bayes´ Theorem

$$P(M \mid A) = \frac{P(M) \cdot P(A \mid M)}{P(M) \cdot P(A \mid M) + P(M^c) \cdot P(A \mid M^c)}$$

The Law of Total Probability

Step 2: Substitute values

$$P(M \mid A) = \frac{0.80 \cdot 0.30}{0.80 \cdot 0.30 + 0.20 \cdot 0.10}$$

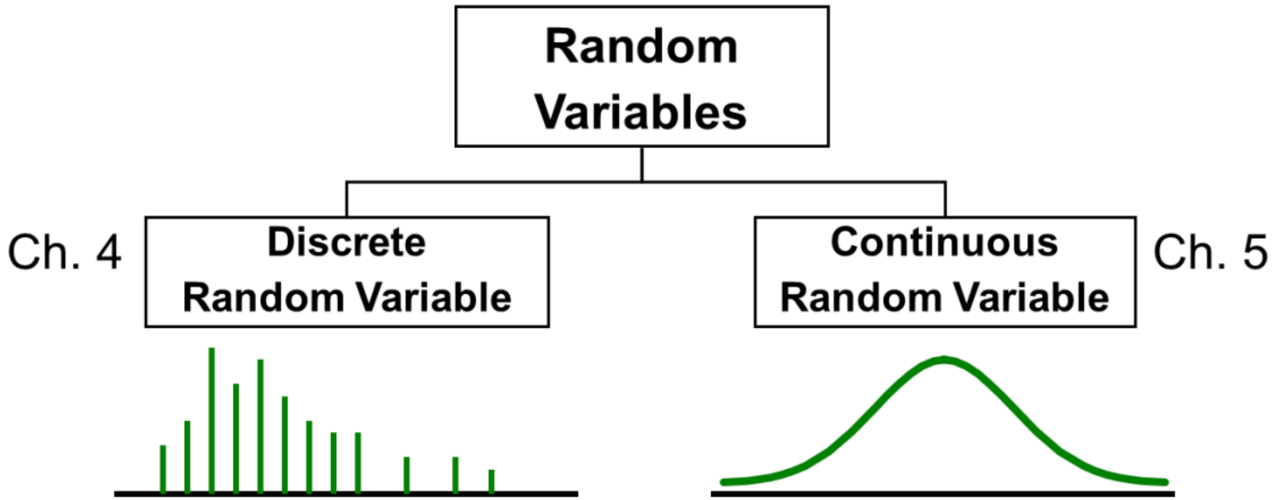
$$= \frac{0.24}{0.24 + 0.02} = \frac{0.24}{0.26} \approx 0.923$$

There is about a **92.3% probability** that a professor who adopts the book had received the advertising material.

LECTURE 4: DISCRETE RANDOM VARIABLES

RANDOM VARIABLE

- **Random Variable**
  - Represents a possible numerical value from a random experiment

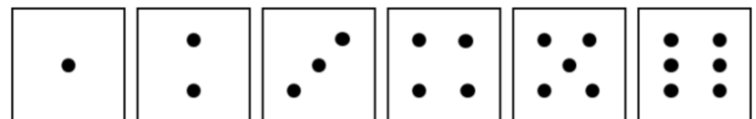


Newbold et al (2013)

DISCRETE RANDOM VARIABLE

- Takes on no more than a countable number of values

Examples:



- Roll a die twice

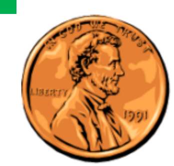
  Let *X* be the number of times 4 comes up (then *X* could be 0, 1, or 2 times)

  x = 0, 1, 2

- **Toss a coin 5 times.**

  Let *X* be the number of heads

  x = 0, 1, 2, 3, 4, 5



Newbold et al (2013)

CONTINUOUS RANDOM VARIABLE

- Can take on any value in an interval
  - Possible values are measured on a continuum

Examples:

- **Weight of packages filled by a mechanical filling process**
- **Temperature of a cleaning solution**
- **Time between failures of an electrical component**

Newbold et al (2013)

PROBABILITY DISTRIBUTION FUNCTION (PDF) FOR DISCRETE RANDOM VARIABLES

Let X be a discrete random variable and x be one of its possible values

- The probability that random variable X takes specific value x is denoted $P(X = x)$
- The probability distribution function of a random variable is a representation of the probabilities for all the possible outcomes.
  - Can be shown algebraically, graphically, or with a table

Newbold et al (2013)

Note: For discrete random variables, the "Probability Distribution Function (PDF)" is more precisely called the **Probability Mass Function (PMF)**. The term "PDF" is often used generically, but "PMF" is the correct technical expression in this case.

PDF OF A DISCRETE RANDOM VARIABLE: TABULAR AND GRAPHICAL REPRESENTATION

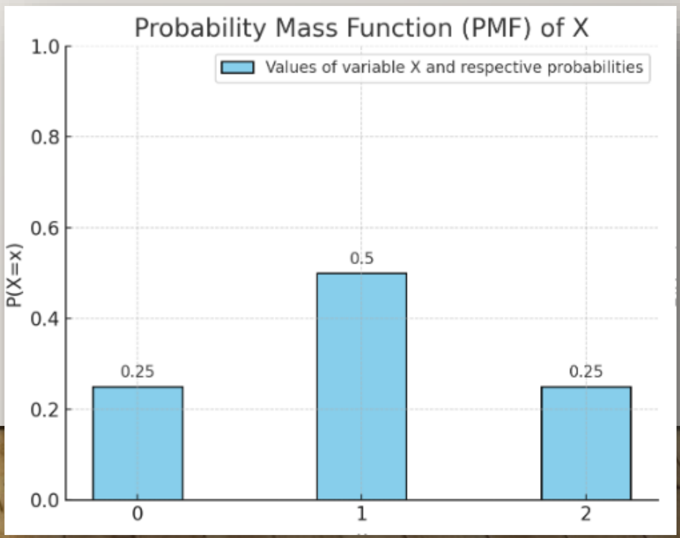
Table of the PMF (or PDF) of a Discrete Random Variable X

Values of the variable X and respective probabilities.

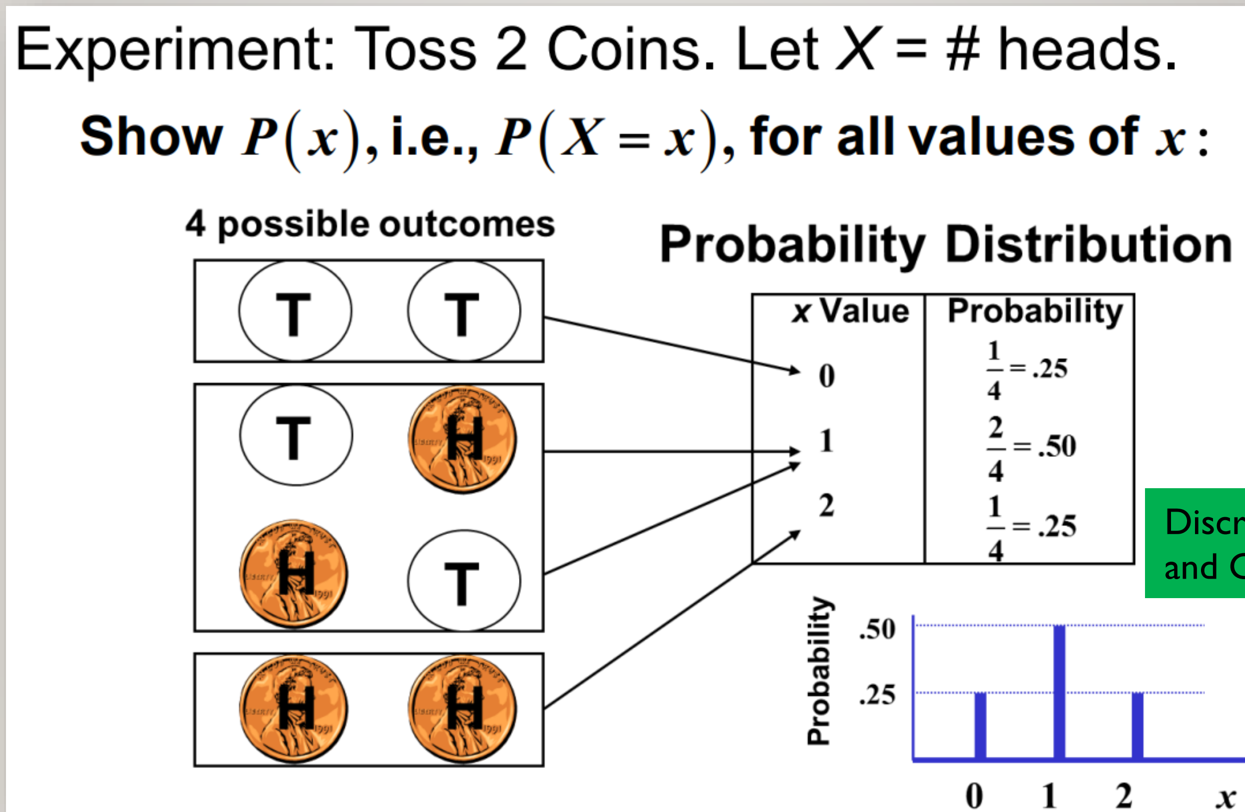
x	x_1	x_2	…	x_n
$f(x)=P(X=x)$	$P(X=x_1)$	$P(X=x_2)$	…	$P(X=x_n)$

Example: Discrete PDF – Table and Graph

x	0	1	2
$P(X=x)$	0.25	0.5	0.25



PDF FOR A DISCRETE RANDOM VARIABLE: OTHER EXAMPLE



Discrete PDF – Table and Graph

Newbold et al (2013)

PROPERTIES REQUIRED OF A PDF

- $\boxed{0 \leq P(x) \leq 1}$ for any value of *x*

- The individual probabilities sum to 1;

$$\boxed{\sum_x P(x) = 1}$$

(The notation indicates summation over all possible *x* values)

Newbold et al (2013)

Summary

$$\boxed{0 \leq P(X = x_i) \leq 1 \quad \text{and} \quad \sum_{i=1}^{n} P(X = x_i) = 1}$$

CUMULATIVE PROBABILITY FUNCTION (CDF)

- The cumulative probability function, denoted $F(x_0)$, shows the probability that *X* does not exceed the value x_0

$$\boxed{F\left(x_0\right)=P\left(X \le x_0\right)}$$

Where the function is evaluated at all values of x_0

Newbold et al (2013)

PDF AND CDF OF A DISCRETE RANDOM VARIABLE: EXAMPLE 1

Example: Toss 2 Coins. Let X = # heads.

x Value	$P(x)$	$F(x)$
0	0.25	0.25
1	0.50	0.75
2	0.25	1.00

Newbold et al (2013)

PDF: $f(x) = P(x) = P(X = x)$

CDF: $F(x) = P(X \leq x)$

PDF OF A DISCRETE VARIABLE: EXAMPLE 2

Let X be the **number of heads obtained in 3 coin tosses**.

- Possible values: $X = 0, 1, 2, 3$
- Type of variable: **discrete random variable**

Discrete PDF: Table and Graph

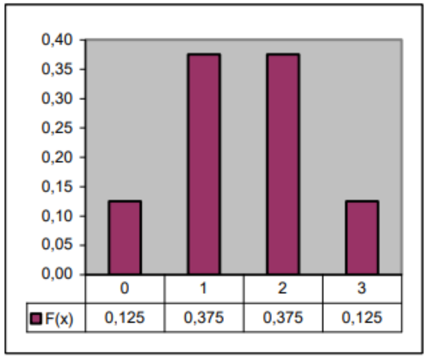
$P[X = 0] = P[(C \cap C \cap C)] = \frac{1}{8}$

$P[X = 1] = P[(F \cap C \cap C) \cup (C \cap F \cap C) \cup (C \cap C \cap F)] = \frac{3}{8}$

$P[X = 2] = P[(F \cap F \cap C) \cup (F \cap C \cap F) \cup (C \cap F \cap F)] = \frac{3}{8}$

$P[X = 3] = P[(F \cap F \cap F)] = \frac{1}{8}$

x	0	1	2	3
f(x)	1/8	3/8	3/8	1/8



Elementos de Estatística e Probabilidades II (uevora.pt)

CDF OF A DISCRETE VARIABLE: EXAMPLE 2

$F(0) = P[X \leq 0] = \sum_{x \leq 0} f(x) = \frac{1}{8};$

$F(1) = P[X \leq 1] = \sum_{x \leq 1} f(x) = \frac{4}{8}$

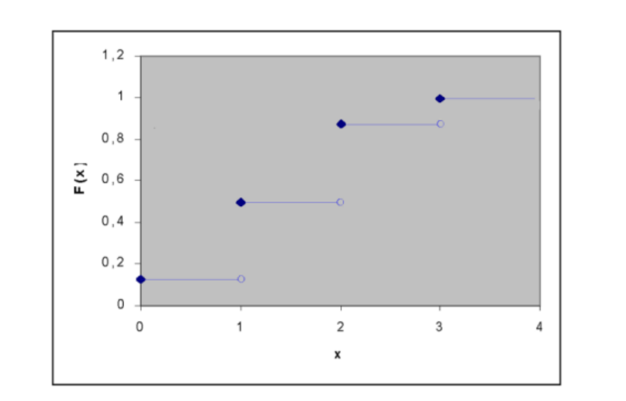
$F(2) = P[X \leq 2] = \sum_{x \leq 2} f(x) = \frac{7}{8}$

$F(3) = P[X \leq 3] = \sum_{x \leq 3} f(x) = 1$

A representação gráfica da função distribuição de uma variável aleatória é "em escada"

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{8}, & 0 \leq x < 1 \\ \frac{4}{8}, & 1 \leq x < 2 \\ \frac{7}{8}, & 2 \leq x < 3 \\ 1, & x \geq 3 \end{cases}$$

Discrete CDF: Table and Graph



Elementos de Estatística e Probabilidades II (uevora.pt)

RELATIONSHIP BETWEEN PDF AND CDF

The derived relationship between the probability distribution and the cumulative probability distribution

- Let X be a random variable with probability distribution $P(x)$ and cumulative probability distribution $F(x_0)$. Then

$$\boxed{F(x_0) = \sum_{x \le x_0} P(x)}$$

(the notation implies that summation is over all possible values of x that are less than or equal to x_0)

Newbold et al (2013)

PROPERTIES REQUIRED OF A CDF

Derived properties of cumulative probability distributions for discrete random variables

- Let X be a discrete random variable with cumulative probability distribution $F(x_0)$. Then

1. $\boxed{0 \leq F(x_0) \leq 1}$ for every number x_0
2. $\boxed{\text{For } x_0 < x_1, \text{ then } F(x_0) \leq F(x_1)}$

Newbold et al (2013)

EXPECTED VALUE OF A DISCRETE RANDOM VARIABLE

- Expected Value (or mean) of a discrete random variable *X*:

$$\boxed{E[X] = \mu = \sum_x xP(x)}$$

PDF: f(x) = P(x) = P(X = x)

- Example: Toss 2 coins,

  x = # of heads,

  compute expected value of *x*:

x	*P*(*x*)
0	.25
1	.50
2	.25

$$\boxed{\begin{aligned} E(x) &= (0 \times .25) + (1 \times .50) + (2 \times .25) \\ &= 1.0 \end{aligned}}$$

Newbold et al (2013)

VARIANCE AND STANDARD DEVIATION OF A DISCRETE RANDOM VARIABLE

- Variance of a discrete random variable *X*

$$\boxed{\sigma^2 = E\left[(X-\mu)^2\right] = \sum_x (x-\mu)^2 P(x)}$$

Can also be expressed as

$$\boxed{\sigma^2 = E\left[X^2\right] - \mu^2 = \sum_x x^2 P(x) - \mu^2}$$

König's Theorem

$$\mathrm{Var}(X) = \sigma^2 = E(X^2) - [E(X)]^2$$

- Standard Deviation of a discrete random variable *X*

$$\boxed{\sigma = \sqrt{\sigma^2} = \sqrt{\sum_x (x-\mu)^2 P(x)}}$$

Newbold et al (2013)

STANDARD DEVIATION EXAMPLE

- Example: Toss 2 coins, X = # heads, compute standard deviation (recall $E[X]=1$)

PDF: P(x) = P(X = x)

x	$P(x)$
0	.25
1	.50
2	.25

Expected Value

E(X) = μ = 1

$$\sigma = \sqrt{\sum_x (x-\mu)^2 P(x)}$$

$$\sigma = \sqrt{(0-1)^2(.25)+(1-1)^2(.50)+(2-1)^2(.25)} = \sqrt{.50} = .707$$

Possible number of heads = 0, 1, or 2

Newbold et al (2013)

FUNCTIONS OF RANDOM VARIABLES

- If $P(x)$ is the probability function of a discrete random variable *X*, and $g(X)$ is some function of *X*, then the expected value of function *g* is

$$\boxed{E\left[g(X)\right]=\sum_{x} g(x)P(x)}$$

Newbold et al (2013)

Example: Let $g(X) = 2X$

$$E[2X] = \sum_{x} 2x \cdot P(X = x)$$

Explanation: Multiply each value of X by 2, then weight by its probability.

LINEAR FUNCTIONS OF RANDOM VARIABLES

- Let random variable *X* have mean μ_x and variance $\sigma^2{}_x$
- Let *a* and *b* be any constants.
- Let $Y = a + bX$
- Then the mean and variance of *Y* are

$$\boxed{\mu_Y = E(a+bX) = a + b\mu_x}$$

$$\boxed{\sigma^2{}_Y = Var(a+bX) = b^2\sigma^2 x}$$

- so that the standard deviation of *Y* is

$$\boxed{\sigma_Y = |b|\sigma_x}$$

Linear Transformation Example: Let *Y* = 3+2*X*

$$\mu_Y = E(Y) = E(3+2X) = 3 + 2E(X) = 3 + 2\mu_X$$

$$\sigma_Y^2 = \mathrm{Var}(Y) = \mathrm{Var}(3+2X) = 2^2\mathrm{Var}(X) = 4\sigma_X^2$$

$$\sigma_Y = \sqrt{\sigma_Y^2} = \sqrt{4\sigma_X^2} = 2\sigma_X$$

Newbold et al (2013)

PROPERTIES OF LINEAR FUNCTIONS OF RANDOM VARIABLES

- Let *a* and *b* be any constants.
- a) $\boxed{E(a) = a \qquad \text{and} \qquad Var(a) = 0}$

  i.e., if a random variable always takes the value *a*, it will have mean *a* and variance 0

Example: Let $a = 2$

$$E(a) = E(2) = 2 = \mu_a$$

$$\mathrm{Var}(a) = \mathrm{Var}(2) = 0 = \sigma_a^2$$

- b) $\boxed{E(bX) = b\mu_x \qquad \text{and} \qquad Var(bX) = b^2\sigma^2{}_x}$

  i.e., the expected value of $b \cdot X$ is $b \cdot E(x)$

Example: Let $Y = 3X$

$$E(Y) = E(3X) = 3E(X) = 3\mu_X$$

$$\mathrm{Var}(Y) = \mathrm{Var}(3X) = 3^2\mathrm{Var}(X) = 9\sigma_X^2$$

$$\sigma_Y = \sqrt{\mathrm{Var}(Y)} = 3\sigma_X$$

Newbold et al (2013)

EXERCISE 4.16

4.16 Given the probability distribution function:

x	0	1	2
Probability	0.25	0.50	0.25

a. Graph the probability distribution function.
b. Calculate and graph the cumulative probability distribution.
c. Find the mean of the random variable X.
d. Find the variance of X.

Newbold et al (2013)

EXERCISE 4.16 A): SOLUTION

✅ Answer:

Given Data

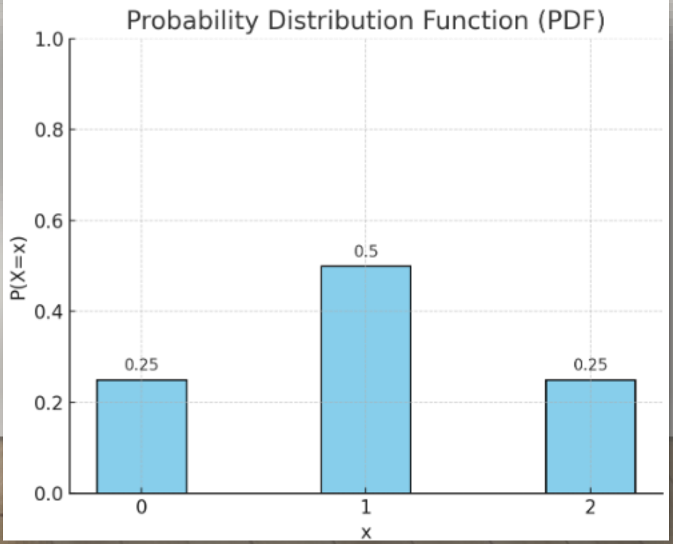
x	0	1	2
P	0.25	0.50	0.25

Let X be the random variable.

Part (a): Graph the probability distribution function

The **probability distribution function (PDF)** is:

$$P(X=0)=0.25, \quad P(X=1)=0.50, \quad P(X=2)=0.25$$



Probability Distribution Function (PDF) – shows the probability for each number of returns.

EXERCISE 4.16 B): SOLUTION

✅ Answer:

Given Data

x	0	1	2
P	0.25	0.50	0.25

Let X be the random variable.

Part (b): Cumulative probability distribution

The **cumulative distribution function (CDF)** is:

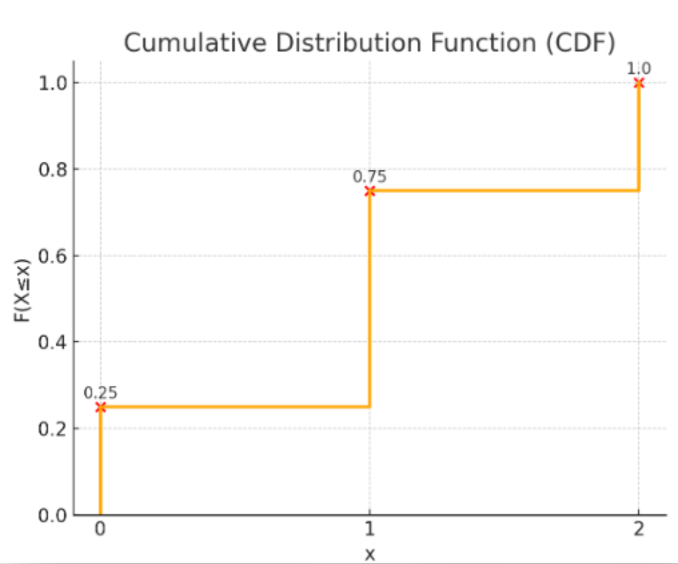
$$F(X) = P(X \leq x)$$

Step by step:

- $F(0) = P(X \leq 0) = 0.25$
- $F(1) = P(X \leq 1) = 0.25 + 0.50 = 0.75$
- $F(2) = P(X \leq 2) = 0.25 + 0.50 + 0.25 = 1.0$

CDF table:

x	0	1	2
F	0.25	0.75	1.00



Cumulative Distribution Function (CDF) – shows the cumulative probability as the number of returns increases.

EXERCISE 4.16 C): SOLUTION

✅ Answer:

Given Data

x	0	1	2
P	0.25	0.50	0.25

Let X be the random variable.

Part (c): Mean of X

$$\mu_X = E(X) = \sum x \cdot P(X = x)$$

$$\mu_X = 0 \cdot 0.25 + 1 \cdot 0.50 + 2 \cdot 0.25 = 0 + 0.50 + 0.50 = 1.0$$

✅ Mean $\mu_X = 1.0$

EXERCISE 4.16 D): SOLUTION

✅ Answer:

Given Data

x	0	1	2
P	0.25	0.50	0.25

Let X be the random variable.

Part (d): Variance of X

$$\sigma_X^2 = E[(X-\mu)^2] = \sum (x-\mu)^2 P(X=x)$$

Step by step:

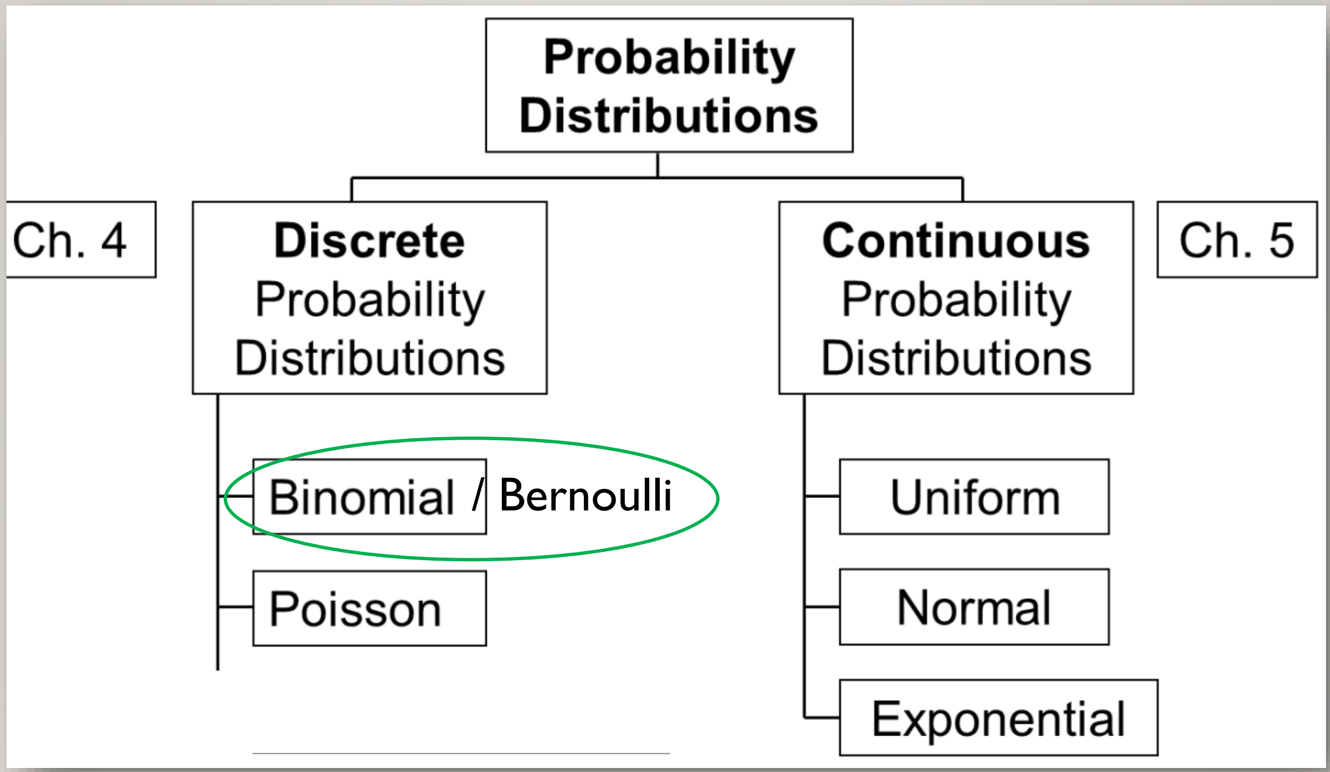
- $x = 0 : (0-1)^2 \cdot 0.25 = 1 \cdot 0.25 = 0.25$
- $x = 1 : (1-1)^2 \cdot 0.50 = 0 \cdot 0.50 = 0$
- $x = 2 : (2-1)^2 \cdot 0.25 = 1 \cdot 0.25 = 0.25$

$$\sigma_X^2 = 0.25 + 0 + 0.25 = 0.50$$

✅ Variance $\sigma_X^2 = 0.50$

LECTURE 4: BINOMIAL DISTRIBUTION

PROBABILITY DISTRIBUTIONS



Newbold et al (2013)

A **Bernoulli random variable** represents a single trial (n = 1) of a Binomial experiment. Therefore, the Bernoulli distribution is a particular case of the **Binomial distribution**.

BERNOULLI DISTRIBUTION

- Consider only two outcomes: "success" or "failure"
- Let P denote the probability of success
- Let $1 - P$ be the probability of failure
- Define random variable X:

$$\boxed{x = 1 \text{ if success, } x = 0 \text{ if failure}}$$

- Then the Bernoulli probability distribution is

$$\boxed{P(0) = (1 - P) \quad \text{and} \quad P(1) = P}$$

For a **Bernoulli distribution**:
- $n = 1$(number of trials)
- $p = P(Success) = P(X = 1)$

X – number of successes in a single trial (x = 0 or 1)

Newbold et al (2013)

PDF of the Bernoulli Distribution: Table and Algebraic Expression

x	1	0
$P(X=x)$	p	$1 - p$

$$P(X = x) = p^x(1 - p)^{1-x}, \quad x = 0, 1$$

The random variable X follows a Bernoulli distribution with parameter p: **X ~ Bernoulli (p)**

MEAN AND VARIANCE OF A BERNOULLI RANDOM VARIABLE

- The mean is $\boxed{\mu_x = P}$

$$\mu_x = E[X] = \sum_x xP(x) = (0)(1-P) + (1)P = P$$

- The variance is $\boxed{\sigma_x^2 = P(1-P)}$

$$\sigma_x^2 = E\left[(X-\mu_x)^2\right] = \sum_x (x-\mu_x)^2 P(x)$$

$$= (0-P)^2(1-P) + (1-P)^2 P = P(1-P)$$

Summary

E(X) = p

Var(X) = p(1 – p)

Newbold et al (2013)

BERNOULLI DISTRIBUTION: EXAMPLE

Bernoulli Distribution: Example with a Die

Let X be the random variable defined as:

$$X = \begin{cases} 1 & \text{if face 5 occurs} \\ 0 & \text{otherwise} \end{cases}$$

Since the die is fair:

$$\boxed{P(X=1) = p = \tfrac{1}{6}, \quad P(X=0) = 1 - p = \tfrac{5}{6}}$$

Therefore:

$$\boxed{X \sim \text{Bernoulli}\left(\tfrac{1}{6}\right)}$$

PDF of the Bernoulli Distribution: Table and Algebraic Expression

X	1	0
$P(X=x)$	1/6	5/6

$$P(X=x) = \left(\frac{1}{6}\right)^x \left(1 - \frac{1}{6}\right)^{1-x} = \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{1-x}, \quad x = 0, 1$$

COMBINATIONS

Combinations ("n choose x")

A **combination** counts the number of ways to **choose x elements from n distinct elements, without considering the order**.

- Notation:

$$\boxed{\binom{n}{x} = \frac{n!}{x!(n-x)!}}$$

Where $n!$ (n factorial) = $n \cdot (n-1) \cdot \cdots \cdot 1$

- **Key point:** Order does **not** matter.

Example: Combinations

Question: How many groups of 2 elements can we form from 3 elements A, B, C?

- Number of elements chosen at a time: $x = 2$
- The combinations of 2 elements from 3 are:

$$AB, AC, BC$$

- Formula:

$$\boxed{\binom{n}{x} = \frac{n!}{x!(n-x)!} = \binom{3}{2} = \frac{3!}{2!(3-2)!} = 3}$$

Answer: There are 3 groups.

DEVELOPING THE BINOMIAL DISTRIBUTION

- The number of sequences with *x* successes in *n* independent trials is:

$$\boxed{C_x^n = \frac{n!}{x!(n-x)!}}$$

Where $n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 1$ and $0! = 1$

- These sequences are mutually exclusive, since no two can occur at the same time

Newbold et al (2013)

BINOMIAL DISTRIBUTION: CHARACTERISTICS

- $\boxed{\text{A fixed number of observations, } n}$
  - e.g., 15 tosses of a coin; ten light bulbs taken from a warehouse
- $\boxed{\text{Two mutually exclusive and collectively exhaustive categories}}$
  - e.g., head or tail in each toss of a coin; defective or not defective light bulb
  - Generally called "success" and "failure"
  - Probability of success is *P*, probability of failure is 1 − *P*
- $\boxed{\text{Constant probability for each observation}}$
  - e.g., Probability of getting a tail is the same each time we toss the coin
- $\boxed{\text{Observations are independent}}$
  - The outcome of one observation does not affect the outcome of the other

Newbold et al (2013)

For a Binomial Distribution:

- n– Number of trials (fixed)
- Two possible outcomes per trial: Success or Failure
- $p = \mathrm{P(Success)}$ – Probability of success (same for each trial)
- Trials are independent

X– Number of successes in n trials, $\mathrm{x} = 0,1, \dots, n$

POSSIBLE BINOMIAL DISTRIBUTION SETTINGS

- A manufacturing plant labels items as either defective or acceptable
- A firm bidding for contracts will either get a contract or not
- A marketing research firm receives survey responses of “yes I will buy” or “no I will not”
- New job applicants either accept the offer or reject it

Newbold et al (2013)

PDF OF A BINOMIAL DISTRIBUTION

$$\boxed{P(x) = \frac{n!}{x!(n-x)!} P^x (1-P)^{n-x}} \Leftrightarrow P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \ldots, n$$

$P(x)$ = probability of ***x*** successes in ***n*** trials, with probability of success ***P*** on each trial

x = number of 'successes' in sample, $(x = 0,\ 1,\ 2,\ \ldots,\ n)$

n = sample size (number of independent trials or observations)

P = probability of "success"

The random variable X follows a Binomial distribution with parameters n and p : ***X*** **~ Binomial (n, p)**

Newbold et al (2013)

Each Bernoulli trial counts as **one experiment**, while the Binomial counts **the number of successes in multiple trials.**

BINOMIAL DISTRIBUTION: EXAMPLE 1

Example: Flip a coin four times, let *x* = # heads:

$$\boxed{\begin{aligned} n &= 4 \\ P &= 0.5 \end{aligned}}$$

$$1 - P = (1 - 0.5) = 0.5$$

$$x = 0,\ 1,\ 2,\ 3,\ 4$$

***X* ~ Binomial (4, 0.5)**

What is the probability of one success in five observations if the probability of success is 0.1?

$x = 1$, $n = 5$, and $P = 0.1$

$$\begin{aligned} P(x=1) &= \frac{n!}{x!(n-x)!} P^x (1-P)^{n-x} \\ &= \frac{5!}{1!(5-1)!} (0.1)^1 (1-0.1)^{5-1} \\ &= (5)(0.1)(0.9)^4 \\ &= .32805 \end{aligned}$$

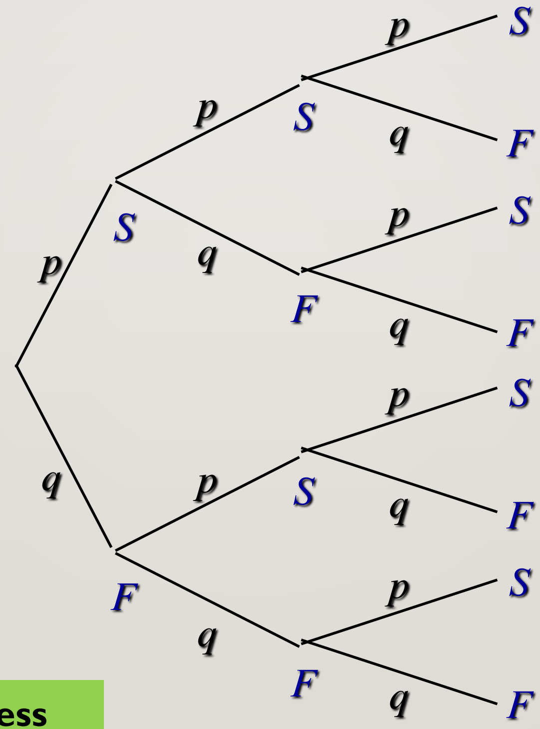
Newbold et al (2013)

BINOMIAL DISTRIBUTION: EXAMPLE 2

Example with a Die:
A fair die is rolled 3 times. Let X be the random variable:

X = *number of successes (face 5) in 3 independent roll of a fair die*

- Possible values: $x = 0,1,2,3$
- Each roll is independent, with probability of success (face 5) equal to $p = \frac{1}{6}$.

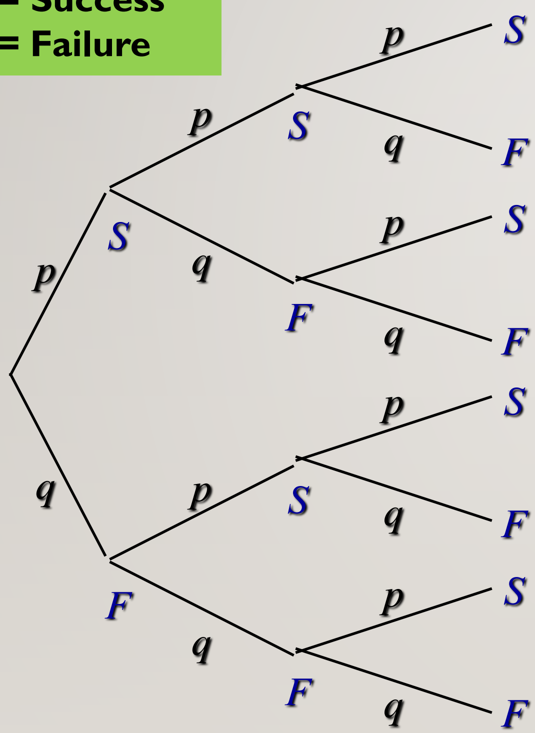


S	Prob	X
(SSS)	p^3	3
(SSF)	p^2q	2
(SFS)	p^2q	2
(SFF)	pq^2	1
(FSS)	p^2q	2
(FSF)	pq^2	1
(FFS)	pq^2	1
(FFF)	q^3	0

S = Success
F = Failure

BINOMIAL DISTRIBUTION: EXAMPLE 2

S = Success
F = Failure



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(SSS)	p^3	3
(SSF)	p^2q	2
(SFS)	p^2q	2
(SFF)	pq^2	1
(FSS)	p^2q	2
(FSF)	pq^2	1
(FFS)	pq^2	1
(FFF)	q^3	0

X ~ Binomial (3, 1/6)

Binomial Probabilities for
$n = 3$ and $P(Success) = p = 1/6$

Number of Successes	Probability
0	q^3
1	$3pq^2$
2	$3p^2q$
3	p^3

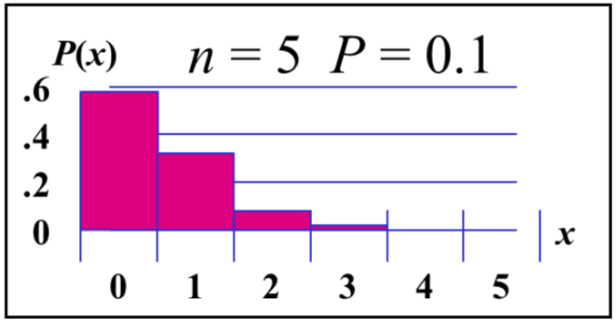
We can express the PMF of a discrete random variable either by listing the probabilities in a table or by using the mathematical formula.

Probability Mass Function (PMF):

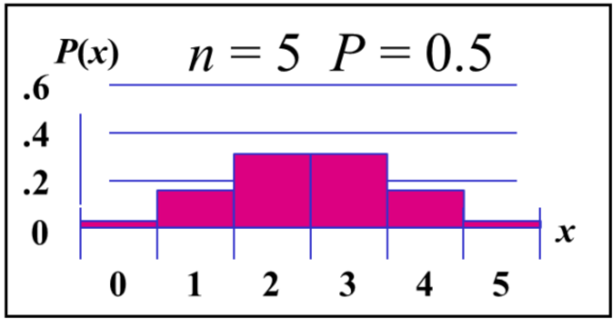
$$P(X = k) = \binom{3}{k} \left(\tfrac{1}{6}\right)^k \left(\tfrac{5}{6}\right)^{3-k}, \quad k = 0, 1, 2, 3$$

SHAPE OF BINOMIAL DISTRIBUTION

- The shape of the binomial distribution depends on the values of *P* and *n*



- Here, *n* = 5 and *P* = 0.1



- Here, *n* = 5 and *P* = 0.5

Newbold et al (2013)

MEAN AND VARIANCE OF A BINOMIAL DISTRIBUTION

- Mean

$$\boxed{\mu = E(x) = nP}$$

- Variance and Standard Deviation

$$\boxed{\begin{aligned}\sigma^2 &= nP(1-P)\\ \sigma &= \sqrt{nP(1-P)}\end{aligned}}$$

Where n = sample size

P = probability of success

$(1 - P)$ = probability of failure

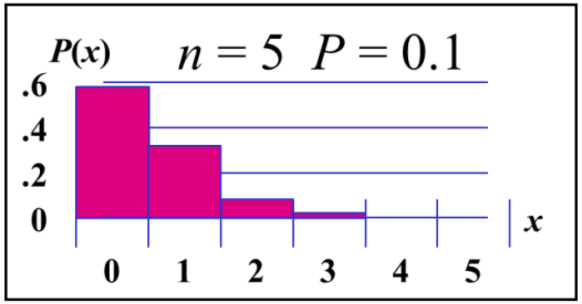
Newbold et al (2013)

BINOMIAL CHARACTERISTICS

Examples

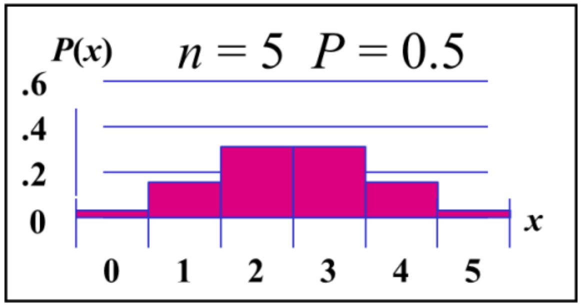
$$\mu = nP = (5)(0.1) = 0.5$$

$$\sigma = \sqrt{nP(1-P)} = \sqrt{(5)(0.1)(1-0.1)}$$
$$= 0.6708$$



$$\mu = nP = (5)(0.5) = 2.5$$

$$\sigma = \sqrt{nP(1-P)} = \sqrt{(5)(0.5)(1-0.5)}$$
$$= 1.118$$



Newbold et al (2013)

USING BINOMIAL TABLES

N	x	…	p = .20	p = .25	p = .30	p = .35	p = .40	p = .45	p = .50
10	0	…	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010
	1	…	0.2684	0.1877	0.1211	0.0725	0.0403	0.0207	0.0098
	2	…	0.3020	0.2816	0.2335	0.1757	0.1209	0.0763	0.0439
	3	…	0.2013	0.2503	0.2668	**0.2522**	0.2150	0.1665	0.1172
	4	…	0.0881	0.1460	0.2001	0.2377	0.2508	0.2384	0.2051
	5	…	0.0264	0.0584	0.1029	0.1536	0.2007	0.2340	0.2461
	6	…	0.0055	0.0162	0.0368	0.0689	0.1115	0.1596	0.2051
	7	…	0.0008	0.0031	0.0090	0.0212	0.0425	0.0746	0.1172
	8	…	0.0001	0.0004	0.0014	0.0043	0.0106	**0.0229**	0.0439
	9	…	0.0000	0.0000	0.0001	0.0005	0.0016	0.0042	0.0098
	10	…	0.0000	0.0000	0.0000	0.0000	0.0001	0.0003	0.0010

Examples:

$$n = 10,\ x = 3,\ P = 0.35: \quad P(x = 3 \mid n = 10,\ p = 0.35) = .2522$$

$$n = 10,\ x = 8,\ P = 0.45: \quad P(x = 8 \mid n = 10,\ p = 0.45) = .0229$$

Newbold et al (2013)

HOMEWORK OF LECTURE 4: QUESTIONS

EXERCISE 3.25

3.25 The probability of A is 0.30, the probability of B is 0.40 and the probability of both is 0.30. What is the conditional probability of A given B? Are A and B independent in a probability sense?

Newbold et al (2013)

EXERCISE 3.85

3.85 The Watts New Lightbulb Corporation ships large consignments of lightbulbs to big industrial users. When the production process is functioning correctly, which is 90% of the time, 10% of all bulbs produced are defective. However, the process is susceptible to an occasional malfunction, leading to a defective rate of 50%. If a defective bulb is found, what is the probability that the process is functioning correctly? If a nondefective bulb is found, what is the probability that the process is operating correctly?

Newbold et al (2013)

EXERCISE 4.18

4.18 An automobile dealer calculates the proportion of new cars sold that have been returned a various numbers of times for the correction of defects during the warranty period. The results are shown in the following table.

Number of returns	0	1	2	3	4
Proportion	0.28	0.36	0.23	0.09	0.04

a. Graph the probability distribution function.
b. Calculate and graph the cumulative probability distribution.
c. Find the mean of the number of returns of an automobile for corrections for defects during the warranty period.
d. Find the variance of the number of returns of an automobile for corrections for defects during the warranty period.

Newbold et al (2013)

EXERCISE 4.33

4.33 For a binomial probability distribution with $P = 0.4$ and $n = 20$, find the probability that the number of successes is equal to 9 and the probability that the number of successes is fewer than 7.

Newbold et al (2013)

EXERCISE 4.46

4.46 A campus finance officer finds that, for all parking tickets issued, fines are paid for 78% of the tickets. The fine is $2. In the most recent week, 620 parking tickets have been issued.

a. Find the mean and standard deviation of the number of these tickets for which the fines will be paid.
b. Find the mean and standard deviation of the amount of money that will be obtained from the payment of these fines.

Newbold et al (2013)

THANKS!

Questions?