Resit Exam 2025–2026

Solutions

January 11, 2026

1 Production (3 points)

1.1 TRS (1 point)

Can rewrite the production function as

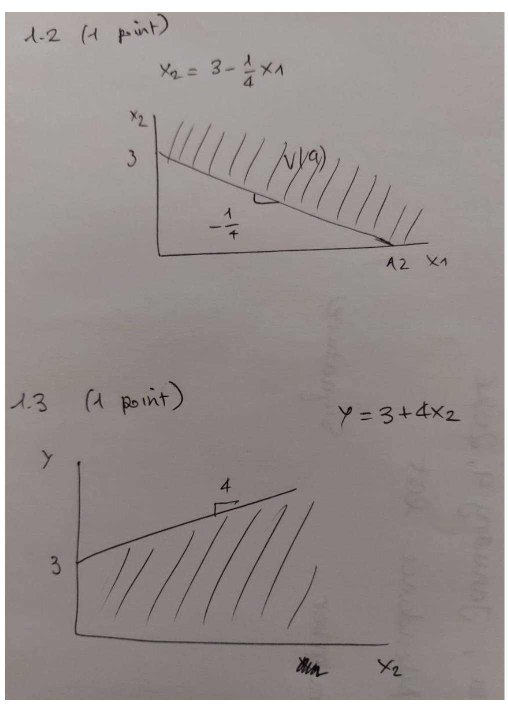
$$y = \beta x_1 + (2 + \gamma) x_2.$$

The marginal rate of technical substitution is

$$\text{TRS} = -\frac{\partial f / \partial x_1}{\partial f / \partial x_2} = -\frac{\beta}{2 + \gamma}.$$

1.2 Isoquant (1 point)

1.3 SR set (1 point)



2 Profit and cost (4 points)

2.1 WACM (1 point) Let w^t be a vector of input prices and x^t be a vector of inputs at time t. Then the Weak Axiom of Cost Minimization (WACM) is given by

$$w^t x^t \leq w^t x^s \quad \text{for all } s, t \text{ such that } y^s \geq y^t.$$

Hence, to test WACM we need the following data over time t:

- input prices w^t,
- input levels x^t,
- output levels y^t.

2.2 Envelope Theorem (1 point) In words, the Envelope Theorem states that when the value of an optimized function changes due to a change in an exogenous variable, only the direct effect of that exogenous variable needs to be considered. This result holds even if the exogenous variable also affects the optimized function indirectly through its effect on an endogenous choice variable.

2.3 Conditional Factor demand

The short-run cost-minimization problem is

$$\min_{x_1} w_1 x_1 + w_2 x_2 \quad \text{s.t.} \quad x_1^\beta k^{1-\beta} = y.$$

Solving for x_1 gives the short-run conditional factor demand:

$$x_1(w, y, x_2 = k) = \left(\frac{y}{k^{1-\beta}}\right)^{1/\beta}$$

With one variable input (x_2 is fixed) and a binding requirement, x_1 is pinned down by technology and output.

2.4 Cost functions

Short-run cost function:

$$c(w, y, x_2 = k) = w_1 \left(\frac{y}{k^{1-\beta}}\right)^{1/\beta} + w_2 k$$

Short-run average variable cost:

$$SAVC(w, y, k) = \frac{c(w, y, k)}{y} = w \, \frac{\left(\frac{y}{k^{1-\beta}}\right)^{1/\beta}}{y} = w \, y^{\frac{1}{\beta}-1} \, k^{-\frac{1-\beta}{\beta}}$$

Short-run average fixed cost:

$$SAFC(w, y, k) = \frac{w_2 k}{y}$$

Short-run marginal cost:

$$MC(w, y, k) = \frac{\partial c(w, y, k)}{\partial y} = w \cdot \frac{1}{\beta} \left(\frac{1}{k^{1-\beta}}\right)^{1/\beta} y^{\frac{1}{\beta}-1} = \frac{w}{\beta} \left(\frac{y}{k}\right)^{\frac{1-\beta}{\beta}}$$

3 Consumer choice (5 points)

3.1 Hicksian downward sloping (1 point)

Convexity of consumer preferences ensures that the Hicksian demand function is downwards sloping. Consider x_1 on the horizontal axis. Then, if the price of good 1 increases, the budget line gets steeper and touches the same indifference curve at a lower amount of x_1. Hence, when the price of good 1 increases, the demand for x_1 decreases while keeping utility constant. This change in consumption due to the change in relative prices is the substitution effect, which the Hicksian demand captures.

3.2 Marshallian Demand (1 point)

(a) Write down the Lagrangian for the UMP.

(b) Take FOCs.

(c) Solve these FOCs for x_1 and x_2 to reach:

$$x_1(p_1, m) = \frac{m}{p_1}\alpha$$

$$x_2(p_2, m) = \frac{m}{p_2}(1-\alpha)$$

3.3 Restrictions on α (1 point)

$$\frac{\partial x_1(p_1, m)}{\partial p_1} = -\frac{m}{p_1^2}\alpha$$

$$\frac{\partial x_2(p_2, m)}{\partial p_2} = -\frac{m}{p_2^2}(1-\alpha)$$

Both goods are ordinary if both derivatives are smaller than zero. Both derivatives are smaller than zero if:

$$\alpha > 0 \quad \text{and} \quad (1-\alpha) > 0 \quad \Rightarrow 0 < \alpha < 1$$

3.4 Indirect utility (1 point)

(a) Fill the Marshallian demand functions into the utility function.

(b) Simplify.

$$v(p, m) = \left(\frac{m}{p_1}\alpha\right)^{\alpha}\left(\frac{m}{p_2}(1-\alpha)\right)^{1-\alpha} = m\,\alpha^{\alpha}(1-\alpha)^{1-\alpha}p_1^{-\alpha}p_2^{\alpha-1}$$

3.5 Lambda (1 point) To obtain λ one needs to take the derivative of the indirect utility function towards m:

$$\frac{\partial v(p,m)}{\partial m} = \lambda(p) = \alpha^{\alpha}(1-\alpha)^{1-\alpha}p_1^{-\alpha}p_2^{\alpha-1}$$

Plug in for $\alpha = \frac{1}{2}$, $p_1 = 1$, and $p_2 = 4$:

$$\lambda(p) = 0.5^{0.5}(1-0.5)^{1-0.5}1^{-0.5}4^{0.5-1} = 0.25$$

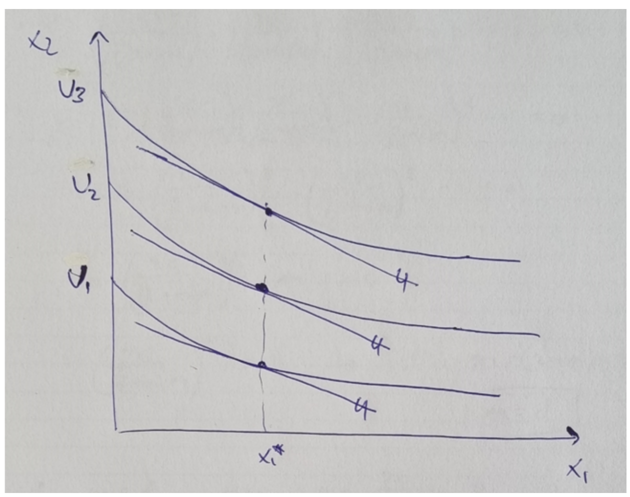
This implies that if a consumer's income increases by 1 (so we relax the constraint by 1), then the consumer can obtain 0.25 additional util.

4 Welfare (4 points)

4.1 Indifference curves (1 point)

The formula for the indifference curve is: $x_2 = u - 2\sqrt{x_1}$.

Drawing this for three levels of u:



Special feature: The varying levels of u just change the intercept of this indifference curve, where the slope of this indifference curve does not depend upon the levels of u. Hence, the indifference curves are parallel: they are vertical shifts from one to the other.

Consequence for income effect: since the slope of this indifference curve does not depend upon the level of u, the indifference curves related to the varying levels of u will be tangent to the budget line for varying income levels at the same level of x_1. This implies that the income effect for x_1 is zero. This is reflected by $x_1^{\star}$ in the figure.

4.2 (2 points)

Consider a consumer with a utility function

$$u = 2\sqrt{x_1} + x_2,$$

income $m = 10$, and prices $p_1 > 0$, $p_2 > 0$.

(a) Write down the Lagrangian for the UMP:

$$\mathcal{L} = 2\sqrt{x_1} + x_2 + \lambda\,(m - p_1 x_1 - p_2 x_2)$$

(a) Take FOCs.

$$\frac{\partial \mathcal{L}}{\partial x_1} = \frac{1}{\sqrt{x_1}} - \lambda p_1 = 0, \qquad \frac{\partial \mathcal{L}}{\partial x_2} = 1 - \lambda p_2 = 0$$

(a) Solve these FOCs for x_1 to reach:

From $1 - \lambda p_2 = 0$ we get $\lambda = \frac{1}{p_2}$. Substituting into the first FOC:

$$\frac{1}{\sqrt{x_1}} = \frac{p_1}{p_2} \quad \Rightarrow \quad \sqrt{x_1} = \frac{p_2}{p_1} \quad \Rightarrow \quad x_1 = \left(\frac{p_2}{p_1}\right)^2.$$

(a) Write down the Lagrangian for the EMP:

$$\mathcal{L} = p_1 x_1 + p_2 x_2 + \mu\,[\,\bar{u} - 2\sqrt{x_1} - x_2\,]$$

(a) Take FOCs.

$$\frac{\partial \mathcal{L}}{\partial x_1} = p_1 - \mu\frac{1}{\sqrt{x_1}} = 0, \qquad \frac{\partial \mathcal{L}}{\partial x_2} = p_2 - \mu = 0$$

(a) Solve these FOCs for x_1 (denoted by h_1) to reach:

From $p_2 - \mu = 0$ we get $\mu = p_2$. Substituting:

$$p_1 = \frac{p_2}{\sqrt{x_1}} \quad \Rightarrow \quad \sqrt{x_1} = \frac{p_2}{p_1} \quad \Rightarrow \quad h_1 = \left(\frac{p_2}{p_1}\right)^2.$$

4.3 CS as exact measure of welfare (1 point)

With quasilinear utility we have that $x_1 = h_1$. The change in consumer surplus is the area to the left of the Marshallian demand curve x_1. The compensating and equivalent variation are the areas to the left of the Hicksian demand curve h_1. Since the Hicksian and Marshalian demand coincide, the compensating and equivalent variation and change in consumer surplus are the same. The equivalent and compensating variation are exact measures of welfare, whereas in general the change in consumer surplus is not. However, since these are equal with quasilinear utility, the change in consumer surplus can be used as an exact measure of welfare.

5 Perfect competition (2 points)

5.1 (1 point)

In long-run perfect competition with free entry and exit, the equilibrium satisfies:

$$Y(p) = X(p) \qquad \text{and} \qquad \pi_i = 0 \ \forall i.$$

1. Derive the firm's supply function $y_i(p)$ for each firm i.

   Marginal cost is

   $$mc(y) = \frac{dc(y)}{dy} = y.$$

   Since the supply curve is given by $mc_i(y) = p$, we obtain

   $$y_i(p) = p.$$

2. Derive market supply, which is the sum over all m firms:

   $$Y(p) = \sum_{i=1}^{m} y_i(p) = \sum_{i=1}^{m} p = mp.$$

3. Use the first condition to find the equilibrium price and firm supply as a function of the number of firms m.

   Market clearing implies

   $$mp = 48 - 2p.$$

   Solving for p,

   $$p = \frac{48}{m+2}.$$

   Thus, individual firm output is

   $$y_i(p) = p = \frac{48}{m+2}.$$

4. Use the second condition to find the number of firms m such that profits are zero.

   Profits are

$$\pi_i = py_i(p) - c_i(y).$$

   Substituting and setting equal to zero,

$$\pi_i = \left(\frac{48}{m+2}\right)^2 - \frac{1}{2}\left(\frac{48}{m+2}\right)^2 - 8 = 0.$$

   This simplifies to

$$\frac{1}{2}\left(\frac{48}{m+2}\right)^2 = 8 \iff \left(\frac{48}{m+2}\right)^2 = 16, \iff \frac{48}{m+2} = 4, \iff m = 10.$$

Hence, in the long run there will be $m = 10$ active firms in this perfectly competitive market.

6 Monopoly (2 points)

6.1 Optimization (1 point) First, obtain inverse demand from demand function:

$$p(Q) = \frac{100 - Q}{2} = 50 - \frac{Q}{2}.$$

Total cost is

$$c(Q) = 50 + 5Q + Q^2 \quad \Rightarrow \quad MC = \frac{\partial TC}{\partial Q} = 5 + 2Q.$$

Marginal revenue:

$$MR = \frac{\partial TR}{\partial Q} = 50 - Q.$$

Profit maximization sets $MR(Q) = MC(Q)$:

$$50 - Q = 5 + 2Q \quad \iff \quad 45 = 3Q \quad \iff \quad Q^\star = 15$$

6.2 Markup(1 point)

The mark-up of a monopolist is defined as the difference between the price and the marginal cost, hence as $p - MC$. The following formula can be derived from the first-order condition (FOC) of the monopolist:

$$\frac{p - MC}{p} = -\frac{1}{\varepsilon}$$

where ε is the elasticity of demand. It is given that $\varepsilon = -3$ and $MC = 10$, so that:

$$\frac{p - 20}{p} = \frac{1}{2}.$$

Solving for p, we obtain $p = 40$ and the mark-up is $40 - 20 = 20$.

The mark-up may be used as a measure of market power since it reflects the ability to set the price above marginal cost, and hence the ability to make profits on the last units sold. In the extreme case, a perfectly competitive firm has no market power, as the mark-up is zero when $p = MC$.