



General Guidelines

- You may use a calculator;
- You may **not** use a programmable calculator;
- You may **not** use notes or books;
- You may have some food and beverages on your desk;
- All other belongings, including phones, must be on the floor;
- You can only leave the room after 30 minutes into the exam and up to 15 minutes before the exam ends;
- Write all your answers on the blank answer sheets brought by you;
- Write your name and student number on every answer sheet;
- Number all your answer sheets and hand them in in chronological order;
- If a question does not ask for an explanation, there is no need to give one;
- Any form of fraud will, at least, imply an invalid grade for this course.

1. Production (3 points)

Let $y = \beta x_1 + \gamma x_2 + 2x_2$ be a production function, where y is the output and x_1 and x_2 are the two inputs.

1.1. Find the Technical Rate of Substitution (TRS) for the production above.

Consider for the following two questions that $\beta = 1$ and $\gamma = 2$. In your answers, make sure to label everything you graph, such as the axes, intercepts, slope, and others.

1.2. Carefully sketch the input requirement set for producing at least 12 units of output:

$$\{ (x_1, x_2) \text{ in } R_+^2 \mid 1x_1 + 2x_2 + 2x_2 \geq 12 \}$$

1.3. Consider that in the short run x_1 is fixed at a value of 3. Carefully sketch the short-run production possibilities set:

$$\{ (y, x_2) \text{ in } R_+^2 \mid 1x_1 + 2x_2 + 2x_2 \geq y, x_1 = 3 \}$$

2. Profit and costs (4 points)

2.1. Provide the formula for the Weak Axiom of Cost Minimization (WACM) and briefly describe the data needed to test it.

2.2. Briefly explain the Envelope theorem. You may use words and/or equations.

For the following two questions, consider that a firm is producing one output y using two inputs, x_1 and x_2 . The production function is $f(x) = x_1^\beta x_2^{1-\beta}$, where factor x_2 is fixed at k in the short-run.

2.3. Now, consider the same firm is cost minimizing. Find the short-run conditional factor demand function $x_1(w, y, x_2 = k)$.

2.4. Find the short-run cost function $c(w, y, x_2 = k)$, as well as the short-run average variable cost, the short-run average fixed cost and the short-run marginal cost.

3. Consumer choice (5 points)

3.1. Which assumption on consumer preferences guarantees that the Hicksian demand curve is downwards sloping. Briefly explain your answer.

Consider that the consumer has a utility function equal to $u = x_1^\alpha x_2^{1-\alpha}$. The consumer has income m , and the price for good x_1 and x_2 are p_1 and p_2 respectively.

3.2. Find the Marshallian demand functions for x_1 and x_2 .

3.3. Under which restriction on α is the x_1 and x_2 derived in question 3.2 ordinary goods?

3.4. Find the indirect utility function.

3.5. To find the Lagrange multiplier lambda one can take the derivative of the indirect utility function towards an exogenous variable. Which exogenous variable is this? Find the Lagrange multiplier via this route. Provide a brief economic interpretation for lambda while assuming that $\alpha = 1/2$, $p_1 = 1$, $p_2 = 4$, and $m = 10$.

4. Welfare (4 points)

Consider a consumer with a utility function equal to $u = 2\sqrt{x_1} + x_2$. The consumer has income $m = 10$, and the price for good x_1 and x_2 are p_1 and p_2 respectively (with $p_1 > 0, p_2 > 0$).

4.1. Carefully sketch three indifference curves with varying levels of utility for the utility function above. Briefly explain the special feature of these indifference curves. Also briefly explain what this special feature implies for the income effect.

4.2. Find both the Marshallian and the Hicksian demand function for good x_1 .

4.3. Use the derived demand functions in question 4.2. to argue that the change in consumer surplus can be used as an exact measure of welfare.

5. Perfect competition (2 points)

Consider a perfectly competitive market. Let the total cost function of a *single* firm be equal to:

$$c(y) = 0.5y^2 + 8,$$

where y is the output. Let the *market* demand be given by:

$$X(p) = 48 - 2p,$$

where p is the price. Suppose that in the long run there is free entry into and exit from the market, and that all potential firms have the same cost function $c(y)$ as above.

5.1. How many firms will be active in this perfectly competitive market in the long run?

6. Monopoly (2 points)

Consider a monopoly with the following demand function and total cost function:

$$Q(p) = 100 - 2p$$
$$c(Q) = 50 + 5Q + Q^2$$

Where Q is the quantity and p is the price.

6.1. What is the monopolist's profit-maximizing level of production?

Consider now another monopolist, with unknown demand and total cost functions.

6.2. This monopolist's elasticity of demand is -2 and its marginal costs are equal to 20. Calculate the mark-up. Briefly explain why the mark-up may be used as a measure of market power.

Scratch paper: