Mathematical Programming I

BSc in Applied Mathematics for Economics and Management (MAEG)

Final Exam

18-05-2026 **Duration: 2h**

Present all calculations and appropriately justify all answers.

1. The DoJo factory produces three products, P_1, P_2, and P_3, using three different types of raw materials, $rm1$, $rm2$, and $rm3$, and aims to determine the production plan that maximizes total profit. To this end, it has formulated the following linear programming problem

$$(P)\left\{\begin{array}{lllll} \max & z = & 10x_1 + 5x_2 + 8x_3 & & (\$) \\ \text{s.t.} & & x_1 + x_2 + 2x_3 & \leq 30 & (rm1) \\ & & x_2 + x_3 & \leq 14 & (rm2) \\ & & 4x_1 + 2x_3 & \leq 48 & (rm3) \\ & & x_1, x_2, x_3 & \geq 0 & \end{array}\right.$$

where x_j measures the production level of P_j ($j = 1, 2, 3$). The optimal table obtained by the simplex method is:

$x_B \backslash$	x_1	x_2	x_3	x_4	x_5	x_6	RHS
x_4	0	0	$\frac{1}{2}$	1	-1	$-\frac{1}{4}$	4
x_2	0	1	1	0	1	0	14
x_1	1	0	$\frac{1}{2}$	0	0	$\frac{1}{4}$	12
$z_{(max)}$	0	0	2	0	5	$\frac{5}{2}$	θ ($)

where x_4, x_5, and x_6 are the slack variables of the first, second, and third constraints, respectively. Answer the following questions.

(a) Write and interpret the optimal solution, the dual solution, and the corresponding optimal value.

(b) State the optimal basis matrix and its inverse.

(c) Perform a sensitivity analysis with respect to c_4 (coefficient of x_4 in the objective function of the given problem written in standard form).

(d) Assume that each ton of unused raw material $rm1$ incurs a storage cost of 5 $u.m.$ ($), that is, $c_4 = 0$ becomes $c_4' = -5$ $u.m.$ ($). Check whether it is necessary to make changes to the optimal production plan and, if necessary, determine the new optimal production plan.

(e) Due to import difficulties, assume that the company now has only 18 tons of raw material $rm1$ and 32 tons of raw material $rm3$ available, with no change in the amount of raw material $rm2$ available. Determine the changes that will occur and indicate the optimal production plan under these conditions.

(f) Suppose the company has the option to purchase additional raw materials. In addition to the original cost, each ton of raw material $rm1$ costs 0,5 $m.u.$, each ton of raw material $rm2$ costs 2 $m.u.$, and each ton of raw material $rm3$ costs 1 $m.u.$. The company can purchase only one of these raw materials.

Without solving a new problem, answer the following questions.

i. Should the company purchase more raw materials?

ii. Which one?

iii. How much?

2. Consider the following Linear Programming (LP) problem:

$$(P)\begin{cases} \max & z = & x_1 & + & 2x_2 & \\ \text{s.a} & & -x_1 & + & x_2 & \leq 5 \\ & & 2x_1 & + & x_2 & \leq 10 \\ & & & & x_2 & \geq 2 \\ & & x_1 & & & \geq 0 \\ & & & & x_2 & \geq 0 \end{cases}$$

(a) Solve it graphically.

(b) Rewrite (P) in standard form and give a feasible basic solution and a non-feasible basic solution.

(c) Add a new constraint to (P) such that a feasible basic solution becomes degenerate. Justify your answer.

(d) Write the dual (D) of (P) and, using the primal-dual complementarity relations, determine the optimal solution of (D).

3. The company DoJo has four industrial sites (L_1, L_2, L_3, and L_4) to which it sends its quality control technicians. Since there are several sections to evaluate at each location, the number of technicians needed at each one varies and is 25, 30, 15, and 10, respectively, for L_1, L_2, L_3, and L_4.

The technicians live in three cities (C_1, C_2, and C_3), from which they will naturally be sent to the four industrial sites. Currently, there are 40, 20, and 25 technicians in C_1, C_2, and C_3, respectively.

The distances in kilometers between the cities and the industrial sites are shown in the following table.

	L_1	L_2	L_3	L_4
C_1	10	13	14	3
C_2	18	5	7	16
C_3	12	16	13	4

Using mathematical programming methods, propose a plan for dispatching the technicians that minimizes the total distance traveled. Justify your choices.
(Hint: if necessary, use the top-left corner method.)

Question	1a	1b	1c	1d	1e	1f	2a	2b	2c	2d	3
Score	2	1	1,5	2	2	2	1,5	1,5	1	2	3,5