Mathematical Programming I

Lisbon School of Economics & Management
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BSc in Applied Mathematics for Economics and Management (MAEG)

Final Exam: solutions

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The solutions are provided in a very brief form. The necessary calculations and justifications are not included in any of the answers.

1. (a) $x^* = (x_1^*, x_2^*, x_3^*, x_4^*, x_5^*, x_6^*) = (12, 14, 0; 4, 0, 0)$; $y^* = (y_1^*, y_2^*, y_3^*, y_4^*, y_5^*, y_6^*) = \left(0, 5, \frac{5}{2}; 0, 0, 2\right)$; $z^* = 190$

   (b) $B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$; $\quad B^{-1} = \begin{bmatrix} 1 & -1 & -\frac{1}{4} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$

   (c) $c_4 \in [-4, 5]$

   (d) Changes need to be made. The new optimal plan corresponds to $x^* = (8, 6, 8, 0, 0, 0)$, $z^* = 174$.

   (e) Changes need to be made. The new optimal plan corresponds to $x^* = (8, 10, 0, 0, 4, 0)$, $z^* = 130$.

   (f) You should purchase another 16 units of rm3.

2. (a) $x^* = \left(\frac{5}{3}, \frac{20}{3}\right)$; $\quad z^* = 15$

   (b) standard form

$$\begin{aligned} \max \quad & z = x_1 + 2x_2 \\ \text{s. a} \quad & -x_1 + x_2 + x_3 = 5, \\ & 2x_1 + x_2 + x_4 = 10, \\ & x_2 - x_5 = 2, \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$

   for example, FBS: $(4, 2, 7, 0, 0)$ UFBS: $(0, 0, 5, 10, -2)$

   (c) For example, when adding the constraint $x_1 - x_2 \leq 2$, the FBS $(4, 2, 7, 0, 0)$ turns into a degenerate FBS.

   (d) dual

$$\begin{aligned} \min \quad & w = 5y_1 + 10y_2 + 2y_3 \\ \text{s. a} \quad & y_1 - 2y_2 \geq 1, \\ & y_1 + y_2 + y_3 \geq 2, \\ & y_1, y_2 \geq 0, \quad y_3 \leq 0. \end{aligned}$$

   primal-dual complementarity relations:

$$\begin{cases} x_1^* \left(y_1^* - 2y_2^* - 1\right) = 0, \\ x_2^* \left(y_1^* + y_2^* + y_3^* - 2\right) = 0, \\ y_1^* \left(-x_1^* + x_2^* - 5\right) = 0, \\ y_2^* \left(2x_1^* + x_2^* - 10\right) = 0, \\ y_3^* \left(x_2^* - 2\right) = 0. \end{cases}$$

   $y^* = (1, 1, 0)$, in the standard form $y^* = (1, 1, 0, 0, 0)$,

3. Send 25 technicians from city 1 to location 1, 10 from city 1 to location 2, 5 from city 1 to location 4, 20 from city 2 to location 2, 15 from city 3 to location 3, and 5 from city 3 to location 4, leaving 5 technicians available in city 3.

   The total distance traveled is 710 km.