Mathematical Programming I

BSc in Applied Mathematics for Economics and Management (MAEG)

Resit Exam

05-06-2026 **Duration: 2h**

Present all calculations and appropriately justify all answers.

1. The company DoJo will produce products P_1, P_2, P_3, and P_4 for the last time, since an essential ingredient required for the production of all four products is no longer being manufactured. The company still has 100 tons of this ingredient in stock. For this purpose, it intends to use at most 200 machine hours and retain its 80 permanent workers.

   With the objective of minimizing production costs, while satisfying the imposed conditions, the following Linear Programming problem was solved:

$$(P)\begin{cases} \min \quad z = & 12x_1 + 15x_2 + 10x_3 + 15x_4 & (\$) \\ \text{s.t.} & 3x_1 + 4x_2 + 2x_3 + 4x_4 = 100 & (\text{ingrediente}) \\ & 6x_1 + x_2 + 4x_3 + 3x_4 \leq 200 & (\text{horas máquina}) \\ & 4x_1 + 5x_2 + 4x_3 + x_4 \geq 80 & (\text{trabalhadores}) \\ & x_1, x_2, x_3, x_4 \geq 0 \end{cases}$$

   where x_j represents the quantity to be produced of P_j (in tons, $j = 1, \ldots, 4$).

   After solving the problem, the following optimal table was obtained:

VB	x_1	x_2	x_3	x_4	x_5	x_6	r_1	r_2	RHS
z (min)	$-\frac{3}{4}$	0	$-\frac{5}{2}$	0	0	0	–	–	375
x_6	$-\frac{1}{2}$	0	$-\frac{3}{2}$	-1	0	1	$\frac{5}{4}$	-1	45
x_5	$\frac{21}{4}$	0	$\frac{7}{2}$	2	1	0	$-\frac{1}{4}$	0	175
x_2	$\frac{3}{4}$	1	$\frac{1}{2}$	1	0	0	$\frac{1}{4}$	0	25

   being x_5 and x_6 the slack variables associated with the second and third constraints, respectively, and r_1 and r_2 the artificial variables associated with the first and third constraints, respectively.

   (a) Present an optimal solution to the dual problem.

   (b) Write a brief report containing all relevant information that can be obtained from the optimal table and from the previous item.

   (c) Indicate the optimal basis matrix and its inverse.

   (d) Does there exist an optimal solution in which the quantity produced of P_2 is 20 tons? If so, provide one.

   (e) For which unit production cost values of P_2 can it be guaranteed that the production plan corresponding to the given table remains optimal?

   (f) Due to a power surge, some machines were damaged, resulting in fewer machine hours being available. Study the consequences of this malfunction.

   (g) DoJo forgot to consider the production of a fifth product. Each ton of this product has a unit production cost of 12, consumes 4 tons of the special ingredient, requires 6 machine hours, and 4 workers. Is it worthwhile to produce this product? If so, determine the new optimal production plan.

(h) Consider that the production costs have been modified, so that the objective function becomes $11x_1 + 12x_2 + 9x_3 + 11x_4$. In addition, there is no longer any limitation on the number of machine hours available; therefore, the second constraint has been removed from the model.

Assuming that the resulting linear programming problem has a unique optimal solution, determine, without solving the problem, the maximum number of different products that can be produced in the optimal solution.

Justify your answer carefully.

2. Consider the following Linear Programming problem:

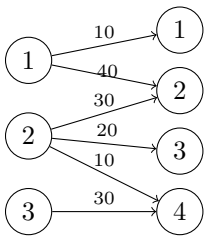
$$(P)\begin{cases} \max \quad z = & -2x_1 + 2x_2 \\ \text{s.t.} & x_1 + x_2 \leq 5 \\ & 2x_2 \geq x_1 \\ & x_1, x_2 \geq 0 \end{cases}$$

(a) Rewrite (P) in standard form and indicate one basic feasible solution and one basic infeasible solution. Justify your selection.

(b) The optimal solution of (P) is $(0, 5)$. Write down the primal-dual complementary slackness conditions and use them to determine the optimal solution of the dual problem of (P).

3. Consider the following table containing the data of a **transportation problem** with three origins and four destinations, and the graph shown alongside.

	1	2	3	4	Supply
1	5	4	3	1	50
2	7	5	2	1	60
3	4	6	3	1	30
Demand	10	70	20	40	



The table shows the unit transportation costs (in 10^3 euros per ton transported) of a certain commodity, as well as the total supply at each origin and the total demand at each destination.

The graph represents a possible transportation plan for this problem.

(a) Verify whether the plan shown in the graph is optimal. If it is not, **starting from the plan represented in the graph**, determine, if possible, the optimal transportation plan. Justify your answer.

(b) The demands and supplies have been modified as follows: customer C1 requires between 5 and 15 units, customer C2 requires at most 70 units, customer C3 requires at least 20 units, and customer C4 requires exactly 40 units.

Assuming that all other data remain unchanged, formulate (write the transportation table of) the new problem as a balanced transportation problem.

Question	1a	1b	1c	1d	1e	1f	1g	1h	2a	2b	3a	3b
Cotação	1,5	2	1	2	1,5	2	1,5	1,5	1	1,5	2,5	2