Mathematical Programming I

Lisbon School of Economics & Management
Universidade de Lisboa

BSc in Applied Mathematics for Economics and Management (MAEG)

Resit Exam: solutions

05-06-2026 **Duration: 2h**

The solutions are provided in a very brief form. The necessary calculations and justifications are not included in any of the answers.

1. (a) $y^* = (y_1^*, y_2^*, y_3^*, y_4^*, y_5^*, y_6^*, y_7^*) = \left(\frac{15}{4}, 0, 0; \frac{3}{4}, 0, \frac{5}{2}, 0\right); w^* = 375;$

   (b) $x^* = (x_1^*, x_2^*, x_3^*, x_4^*, x_5^*, x_6^*) = (0, 25, 0, 0; 175, 45); z^* = 375;$

   (c) $B = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 1 & 1 \\ -1 & 0 & 5 \end{bmatrix}$; $\quad B^{-1} = \begin{bmatrix} \frac{5}{4} & 0 & -1 \\ -\frac{1}{4} & 1 & 0 \\ \frac{1}{4} & 0 & 0 \end{bmatrix}$

   (d) yes, $x^* = (0, 20, 0, 2; 165, 50); z^* = 375;$

   (e) $c_2 \in [0, 15]$, i.e. for any value not greater than the current one.

   (f) If the total number of available hours is greater than or equal to 25, i.e., $b_2 \in [25, +\infty)$, the optimal production plan remains unchanged, and the total cost is also unchanged.

   If it is less than 25, $b_2 \in [0, 25[$, then, by the dual simplex algorithm, the problem is infeasible.

   (g) Yes. The optimal production plan consists of producing only 25 tons of the new product, leaving 50 machine hours unused and employing 20 more workers than the minimum required, for a total cost of 300.

   (h) two

2. (a) standard form:

$$\begin{array}{lll} \max & z = -2x_1 + 2x_2 & \\ \text{s. a} & x_1 + x_2 + x_3 & = 5, \\ & -x_1 + 2x_2 - x_4 & = 0, \\ & x_1, x_2, x_3, x_4 & \geq 0 \end{array}$$

   for example, B.F.S.: $(0, 5, 0, 10)$; N.B.F.S.: $(5, 0, 0, -5)$

   (b) primal-dual complementary slackness conditions:

$$\begin{cases} x_1^* (y_1^* - y_2^* + 2) = 0, \\ x_2^* (y_1^* + 2y_2^* - 2) = 0, \\ y_1^* (5 - x_1^* - x_2^*) = 0, \\ y_2^* (-x_1^* + 2x_2^*) = 0, \end{cases}$$

   $y^* = (2, 0)$, and for the standard form $y^* = (2, 0, 4, 0)$,

3. (a) $x^* = (x_{11}, \ldots, x_{14}, x_{21}, \ldots, x_{24}, x_{31}, \ldots, x_{34})^* = (0, 50, 0, 0, 0, 20, 20, 20, 10, 0, 0, 20)$, the total cost is 420.

   (b) Transportation table corresponding to the balanced problem:

	$C1_{ob}$	$C1_{fac}$	$C2_{fac}$	$C3_{ob}$	$C3_{fac}$	$C4$	CF	a_i
$O1$	5	5	4	3	3	1	0	50
$O2$	7	7	5	2	2	1	0	60
$O3$	4	4	6	3	3	1	0	30
OF	M	0	0	M	0	M	0	155
b_j	5	10	70	20	75	40	75	