4 Integer Linear Programming

1. TBA Airlines is a small air company, specialized in regional flights. The management is considering an expansion and it has the possibility to buy small or medium size airplanes. Find the best strategy, knowing that at the moment no more than two small airplanes can be bought and that \$100 millions are available to invest. Consider also the values in the following table:

	Small airplane	Medium size airplane
Annual profit per airplane	\$1 million	\$5 millions
Cost per airplane	\$5 millions	\$50 millions

2. `Items selection.` An oil company intends to select 5 out of 10 wells: $P1, P2, \ldots, P10$, to which are associated the cost $c_1, c_2, \ldots, c_{10}$, respectively. According to commitments with the local government, the company must comply with the following restrictions for regional development:

   r1) the selection of both P1 and P7 block selection of P8;

   r2) the selection of P3 or P4 block selection of P5;

   r3) from P5, P6, P7 and P8 at most two can be selected;

   r4) the selection of P1 forces selection of P10.

   Formulate the problem and solve it assigning costs at your choice.

3. `Investments selection.` Five investments are being evaluated over a 3-year planning horizon. The following table gives the expected returns for each investment and the associated yearly expenditures:

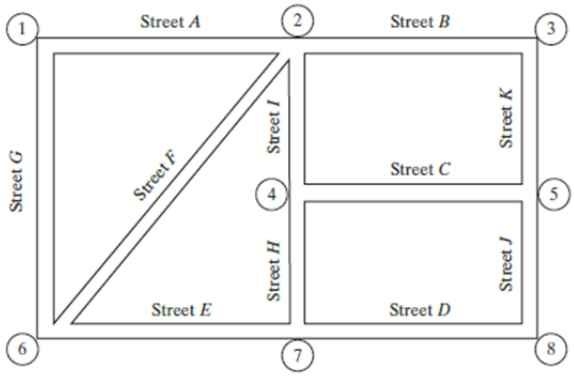
Investment	**Expenditures (\$ million/year)**			**Returns (\$ million)**
	Year 1	**Year 2**	**Year 3**	
1	5	1	8	20
2	4	7	10	40
3	3	9	2	20
4	7	4	1	15
5	8	6	10	30
Available Funds (\$ million)	**25**	**25**	**25**	–

Which investments should be selected over the 3-year horizon?

4. `Fixed Charge Problem - Choosing a Telephone Company.` I have been approached by three telephone companies to subscribe to their service in the United States. MaBell will charge a flat \$16 per month plus \$.25 a minute. PaBell will charge \$25 a month but will reduce the per-minute cost to \$.21. As for BabyBell, the flat monthly charge is \$18, and the cost per min is \$.22. I usually make an average of 200 minutes of calls a month. Assuming that I do not pay the flat monthly fee unless I make calls and that I can apportion my calls among all three companies as I please, how should I use the three companies to minimize my monthly telephone bill?

   Note:This problem can be solved readily without ILP. Nevertheless, it is instructive to formulate it as an integer program.

5. `Set Covering Problem.` To promote on-campus safety, the Safety Department is in the process of installing emergency telephones and other emergency devices at selected locations. The department wants to install the minimum number of these equipements that serve each of the campus main streets. The figure below maps the campus principal streets.



6. `Sequencing Problem.` Jobco uses a single machine to process three jobs. Both the processing time and the due date (in days) for each job are given in the following table. The due dates are measured from zero, the assumed start time of the first job.

Job	Processing time (day)	Due date (day)	Late penalty (\$/day)
1	5	25	19
2	20	22	12
3	15	35	34

   The objective of the problem is to determine the job sequence that minimizes the late penalty for processing all three jobs.

Some solutions

4.1 $x^* = (0, 2),\ z^* = 10.$

4.2 $x^* = (0, 1, 0, 0, 0, 1, 1, 0, 1, 1),\ z^* = 11.$

4.3 $x^* = (1, 1, 1, 1, 0),\ z^* = 95.$

4.4 $x^* = (x_M, x_P, x, y_M, y_P, y_B) = (0, 0, 200, 0, 0, 1),\ z^* = 62.$

4.5 $x^* = (1, 1, 0, 0, 1, 0, 1, 0),\ z^* = 4.$

4.6 $x^* = (t_1, t_2, t_3, y_{12}, y_{13}, y_{23}, s_1^-, s_1^+, s_2^-, s_2^+, s_3^-, s_3^+) = (20, 0, 25, 0, 1, 1, 0, 0, 2, 0, 0, 5),\ z^* = 170.$