# STATISTICAL LABORATORY

## Lecture 4: Frequency Distribution and Measures of Location (Practice)

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# Exercise 3.1

The wages (in monetary units) of 40 factory workers are presented below:

| | | | |
|---|---|---|---|
| 50 | 55 | 85 | 62 |
| 35 | 70 | 75 | 68 |
| 90 | 38 | 72 | 70 |
| 45 | 120 | 50 | 75 |
| 80 | 50 | 48 | 83 |
| 38 | 35 | 45 | 92 |
| 42 | 125 | 150 | 85 |
| 110 | 115 | 70 | 140 |
| 100 | 95 | 75 | 145 |
| 60 | 90 | 155 | 90 |

a) Group the data into 4 classes of equal width, with the lower bound of the first class set at 35 monetary units, and construct the frequency table.

b) Provide a graphical representation of the data.

# Exercise 3.1 (Continued)

c) Present the cumulative frequency function for the class limits and draw the corresponding ogive.

d) Compute the measures of central tendency for these data.

# Exercise 3.1 (Suggested Answers)

a) Frequency table for the wage data

| $I_j$ | $x'_j$ | $n_j$ | $f_j$ | $N_j$ | $F^*_j$ |
|---|---|---|---|---|---|
| $[35, 65]$ | 50 | 14 | 0.350 | 14 | 0.350 |
| $(65, 95]$ | 80 | 17 | 0.425 | 31 | 0.775 |
| $(95, 125]$ | 110 | 5 | 0.125 | 36 | 0.900 |
| $(125, 155]$ | 140 | 4 | 0.100 | 40 | 1.000 |

b) $f_j/h$: $0.4667, 0.5667, 0.1667, 0.1333$ ;

c) $F^*(35) = 0$, $F^*(65) = 0.35$, $F^*(95) = 0.775$, $F^*(125) = 0.9$, $F^*(155) = 1$

# Exercise 3.1 (Suggested Answers)

d) $\bar{x} = \frac{\sum_{j=1}^{m} n_j \cdot x'_j}{n}$

$$\sum_{j=1}^{4} n_j \cdot x'_j = (14 \times 50) + (17 \times 80) + (5 \times 110) + (4 \times 140) = 3170$$

$$\bar{x} = \frac{3170}{40} = 79.25$$

# Exercise 3.1 (Suggested Answers)

Median:

$$Me = l + \frac{0.5 - F^*(l)}{F^*(L) - F^*(l)} \cdot h$$

- Position of median: $\frac{n}{2} = \frac{40}{2} = 20$, which places the median in the interval $(65, 95]$.
- Boundaries: $l = 65$, $L = 95$, and class width $h = 30$.
- Cumulative relative frequencies ($F^*$): $F^*(l) = \frac{14}{40} = 0.35$ and $F^*(L) = \frac{31}{40} = 0.775$.
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$$Me = 65 + \frac{0.5 - 0.35}{0.775 - 0.35} \times 30 = 65 + \frac{0.15}{0.425} \times 30 = 65 + 10.588 = 75.588$$

# Exercise 3.1 (Suggested Answers)

Mode:

$$Mo = l + \frac{f^{**}}{f^{*} + f^{**}} \cdot h$$

notation : $f^{*}$: the class before the modal class, $f^{**}$: the class after the modal class.

$$Mo = 65 + \frac{0.125}{0.35 + 0.125} \times 30 = 65 + \frac{0.125}{0.475} \times 30 = 65 + 7.895 = 72.895$$

# Exercise 3.2

Consider the information provided in exercise 2.3

a) Assess the symmetry (or asymmetry) of the distribution using a boxplot.

b) Determine the new mean salary after a general wage increase of 10%.

c) Evaluate the concentration of the data.

# Exercise 3.2 (Suggested Answers)

a) (600, 988.889, 1065.714, 1159.091, 1300); Negative skewness

b) 1164.9

c) $G = 0.081$

# Exercise 3.3

A sports club measured the weight of its 40 athletes. The observed mean weight was 66.5 kg, the median was 64 kg, and the 1st and 3rd quartiles were 62 kg and 70 kg, respectively. The "extreme" values observed (in kg) were: 40, 51, 53, 53 (on the left tail) and 80, 85, 95, and 101 (on the right tail)

a) Explain the meaning of the 1st and 3rd quartiles in this context.

b) Identify and classify the outliers based on the extreme values.

c) Which measure of central tendency would you consider most appropriate to use? Justify your answer.

# Exercise 3.3 (Suggested Answers)

a) $Q_1 = 62$ ; $Q_3 = 70$.
$Q_1$ indicates that 25% of the athletes in the sports club have a weight equal to or less than 62 kg, while the remaining 75% weigh 62 kg or more.
$Q_3$ indicates that 75% of the athletes have a weight equal to or less than 70 kg, while the remaining 25% weigh 70 kg or more.

b) To find outliers, we first compute the Interquartile Range (IQR):

$$\text{IQR} = Q_3 - Q_1 = 70 - 62 = 8$$

Inner Fences (Moderate Outliers):

- Lower bound: $Q_1 - 1.5 \times \text{IQR} = 62 - 1.5(8) = 62 - 12 = 50$
- Upper bound: $Q_3 + 1.5 \times \text{IQR} = 70 + 1.5(8) = 70 + 12 = 82$

Outer Fences (Severe Outliers):

- Lower bound: $Q_1 - 3.0 \times \text{IQR} = 62 - 3.0(8) = 62 - 24 = 38$
- Upper bound: $Q_3 + 3.0 \times \text{IQR} = 70 + 3.0(8) = 70 + 24 = 94$

Moderate outliers: $40, 85$; Severe outliers: $95, 101$

c) Median or Mode, as they are not affected by extreme values.