Microeconomics

Fall 2026-2027

Mock Midterm – **SOLUTIONS**
October 2026

Duration: 1 hour (60 minutes)

Total: 20 points (Part A: 10 points, Part B: 10 points)

Coordinator: Matilde Cappelletti

General Guidelines

- You may use a calculator;
- You may not use a programmable calculator;
- You may not use notes or books;
- All other belongings, including phones, must be on the floor;
- Write your name and student number on every answer sheet;
- Any form of fraud will, at least, imply an invalid grade for this course.

True / False questions

Each statement is worth 1 point (10 points in total): 1 point for the correct answer with a correct justification, 0.5 points for the correct answer with an incomplete justification, and 0 otherwise. An answer without justification does not receive any marks. Each statement is followed by the correct answer and a short justification.

1. A utility function is cardinal, so the actual numeric values it assigns carry meaningful economic interpretation (e.g., $u(x) = 10$ is "twice as good" as $u(x) = 5$).

False. Utility is ordinal: only the ranking of bundles matters, not the size of the numbers assigned to them.

2. The marginal rate of substitution can be written as $MRS = -MU_1/MU_2$.

True. The MRS is the slope of the indifference curve, obtained from totally differentiating $u(x_1, x_2) = \bar{u}$.

3. A preference relation that is both complete and transitive is called rational.

True. These are exactly the two axioms that define a rational preference relation.

4. Under completeness, transitivity and convexity there exists a continuous utility function that represents those preferences.

False. Continuity of the preference relation is required. Convexity is not needed for a utility representation; it is needed to guarantee the second-order conditions of the maximization problem.

5. At the optimal bundle, the marginal rate of substitution must be greater than the economic rate of substitution.

False. At an interior optimal bundle, $MRS = ERS$ (the slope of the indifference curve equals the slope of the isobudget line).

6. Transitivity requires that if $x \succsim y$ and $y \succsim z$, then $z \succsim x$.

False. Transitivity requires $x \succsim z$, not $z \succsim x$.

7. The Marshallian demand function $x(p, m)$ gives the bundle that minimizes the expenditure needed to reach a target utility level.

False. Marshallian demand maximizes utility subject to the budget constraint. The Hicksian demand is the one that minimizes expenditure for a target utility level.

8. With good 1 on the horizontal axis and good 2 on the vertical axis, the slope of the isobudget line is p_2/p_1.

False. The isobudget line obtained from the budget line $p_1x_1+p_2x_2 = m$, i.e. $x_2 = m/p_2-(p_1/p_2)\,x_1$, has slope $-p_1/p_2$.

9. The MRS captures how much of good 2 a consumer is willing to give up to get one more unit of good 1, while moving along the budget line, holding total spending constant.

False. The MRS is defined moving along an *indifference curve* (keeping utility constant), not along the budget line.

10. With two goods, the Lagrangian for the utility-maximization problem has two unknown endogenous variables and therefore two first-order conditions.

False. There are three unknowns (x_1, x_2 and the multiplier λ), so there are three first-order conditions.

Calculation questions

---

11. Find the partial derivatives f_x and f_y of $f(x,y) = x^2y(6-x-y)$.

Product rule:
$f_x = 2xy(6-x-y) - x^2y = xy(12-3x-2y)$
$f_y = x^2(6-x-y) - x^2y = x^2(6-x-2y)$

12. Let $g(x,y) = xy(12-3x-2y)$.
(a) [0.5] Find all critical points with $x > 0$ and $y > 0$ by solving the system of first-order conditions.
(b) [0.5] Check the second-order conditions and classify the critical point.

(a) $g = 12xy - 3x^2y - 2xy^2$, so
$g_x = 12y - 6xy - 2y^2 = 2y(6-3x-y), \quad g_y = 12x - 3x^2 - 4xy = x(12-3x-4y)$.
With $x, y > 0$ the FOCs reduce to the linear system $3x+y = 6$ (A) and $3x+4y = 12$ (B). Subtracting $(B)-(A)$: $3y = 6$, it follows that $y^* = 2$ and $x^* = \frac{4}{3}$.

(b) $g_{xx} = -6y = -12, \quad g_{yy} = -4x = -\frac{16}{3}, \quad g_{xy} = g_{yx} = 12 - 6x - 4y = -4$.
$g_{xx} < 0$, $g_{yy} < 0$ and $g_{xx}g_{yy} - g_{xy}^2 = 64 - 16 = 48 > 0$, so the point is a local maximum.

13. Find the marginal rate of substitution of $u(x_1,x_2) = x_1^{0.3}x_2^{0.7}$ and evaluate it at the bundle $(x_1,x_2) = (6,14)$.

$$MRS = -\frac{MU_1}{MU_2} = -\frac{0.3}{0.7}\cdot\frac{x_2}{x_1} = -\frac{3}{7}\cdot\frac{x_2}{x_1}$$
At $(6,14)$: $MRS = -\frac{3}{7}\cdot\frac{14}{6} = -1$

14. A consumer has utility $u(x_1,x_2) = x_1^{1/3}x_2^{2/3}$, income $m = 60$ and faces prices $p_1 = 2$ and $p_2 = 4$. Find the optimal bundle.

Solve the Lagrangian. Alternatively, Cobb-Douglas: constant expenditure shares.
$x_1 = \frac{1}{3}\cdot\frac{m}{p_1} = \frac{20}{2} = 10, \quad x_2 = \frac{2}{3}\cdot\frac{m}{p_2} = \frac{40}{4} = 10$
Optimal bundle: $(x_1,x_2) = (10,10)$

15. A consumer has the quasilinear utility function $u(x_1,x_2) = 2\sqrt{x_1} + x_2$, income m and faces prices p_1 and p_2. Set up the Lagrangian and find the Marshallian demand function for good 1 (assume an interior solution).

Lagrangian: $\mathcal{L} = 2\sqrt{x_1} + x_2 - \lambda(p_1x_1 + p_2x_2 - m)$
FOCs: $x_1^{-1/2} = \lambda p_1, \quad 1 = \lambda p_2, \quad p_1x_1 + p_2x_2 = m$
From the second FOC, $\lambda = 1/p_2$. Substituting in the first: $x_1^{-1/2} = p_1/p_2$.
$$x_1(p,m) = \left(\frac{p_2}{p_1}\right)^2$$
$$x_2 = \frac{m}{p_2} - \frac{p_2}{p_1}.$$

16. The indirect utility function is $v(p,m) = \dfrac{m}{2\sqrt{p_1 p_2}}$. Find the Lagrange multiplier λ at $p_1 = 1$, $p_2 = 4$. Which exogenous variable do you differentiate with respect to?

The multiplier is the marginal utility of income, so we differentiate with respect to income m: $\lambda = \dfrac{\partial v}{\partial m}$.

$\lambda = \dfrac{1}{2\sqrt{p_1 p_2}} = \dfrac{1}{2 \cdot 2} = \dfrac{1}{4}$

17. The expenditure function is $e(p,u) = 2\sqrt{p_1 p_2}\, u$. Use Shephard's lemma to find the Hicksian demand $h_1(p,u)$ and evaluate it at $p_1 = 1$, $p_2 = 4$, $u = 10$.

$h_1 = \dfrac{\partial e}{\partial p_1} = \sqrt{\dfrac{p_2}{p_1}}\, u$

$h_1(1,4,10) = \sqrt{4} \cdot 10 = 20$

18. A consumer has the indirect utility function $v(p_1,p_2,m) = \dfrac{m - 2p_1}{2\sqrt{p_1 p_2}}$, defined for $m > 2p_1$. Use Roy's identity to derive the Marshallian demand $x_1(p_1,p_2,m)$. Show the derivatives you take.

Write $v = \frac{1}{2}(m - 2p_1)\, p_1^{-1/2} p_2^{-1/2}$. Then

$\dfrac{\partial v}{\partial m} = \dfrac{1}{2\sqrt{p_1 p_2}}$,

$\dfrac{\partial v}{\partial p_1} = \frac{1}{2}p_2^{-1/2}\Big[-2p_1^{-1/2} - \frac{1}{2}(m-2p_1)p_1^{-3/2}\Big] = \frac{1}{2}p_2^{-1/2}p_1^{-3/2}\Big[-2p_1 - \frac{m}{2} + p_1\Big] = -\frac{1}{4}\, p_2^{-1/2} p_1^{-3/2}\,(2p_1 + m)$.

Roy's identity: $x_1 = -\dfrac{\partial v/\partial p_1}{\partial v/\partial m} = \frac{1}{4}p_2^{-1/2}p_1^{-3/2}(2p_1 + m) \cdot 2(p_1 p_2)^{1/2} = \dfrac{2p_1 + m}{2p_1}$

$x_1(p_1,p_2,m) = 1 + \dfrac{m}{2p_1}$.

19. A consumer has the Marshallian demand function $x_1(p_1,p_2,m) = 1 + \dfrac{m}{2p_1}$. At $p_1 = 1$, $p_2 = 4$ and $m = 20$, use the Slutsky equation, $\partial x_1/\partial p_1 = SE - IE$ with $IE = x_1\, \partial x_1/\partial m$, to find the substitution effect and the income effect. Is good 1 normal or inferior? Ordinary or Giffen?

At $(p_1,p_2,m) = (1,4,20)$: $x_1 = 1 + \dfrac{20}{2} = 11$, $\quad \dfrac{\partial x_1}{\partial p_1} = -\dfrac{m}{2p_1^2} = -10$, $\quad \dfrac{\partial x_1}{\partial m} = \dfrac{1}{2p_1} = \dfrac{1}{2}$.

$IE = x_1 \dfrac{\partial x_1}{\partial m} = 11 \cdot \dfrac{1}{2} = 5.5$.

$SE = \dfrac{\partial x_1}{\partial p_1} + IE = -10 + 5.5 = -4.5$.

Since $\partial x_1/\partial m > 0$ (equivalently $IE > 0$), good 1 is normal. Since $\partial x_1/\partial p_1 < 0$, it is ordinary (not Giffen).

20. A consumer has utility $u = \sqrt{x_1 x_2}$ and income $m = 40$. The price $p_2 = 4$ is fixed and p_1 rises from 1 to 4. Her initial utility is 10. Find the compensating variation.

$CV = e(p^1, v^0) - e(p^0, v^0) = e(p^1, v^0) - m$

From the first two FOCs of the constrained maximization problem: $x_2/x_1 = p_1/p_2$ gives $x_1 = x_2$, so spending is split equally and the Marshallian demands are $x_1 = m/2p_1$ and $x_2 = m/2p_2$, then $v(p,m) = \frac{m}{2\sqrt{p_1 p_2}}$.

Using $u = v(p,m)$ and $e(p,u) = m$, get $e(p,u) = u2\sqrt{p_1 p_2}$

At $p^1 = (4,4)$ $CV = e(p^1, v^0) - m = 10 \cdot 2\sqrt{16} - 40 = 40$