Instituto Superior de Economia e Gestão

Decision Making and Optimization

Exam Prototype 1

Exam Prototype 1 **Duration: 2h**

Notes: -Justify all answers and present the calculations carried out.
-Answer all questions using methodologies taught in Decision Making & Optimization classes.

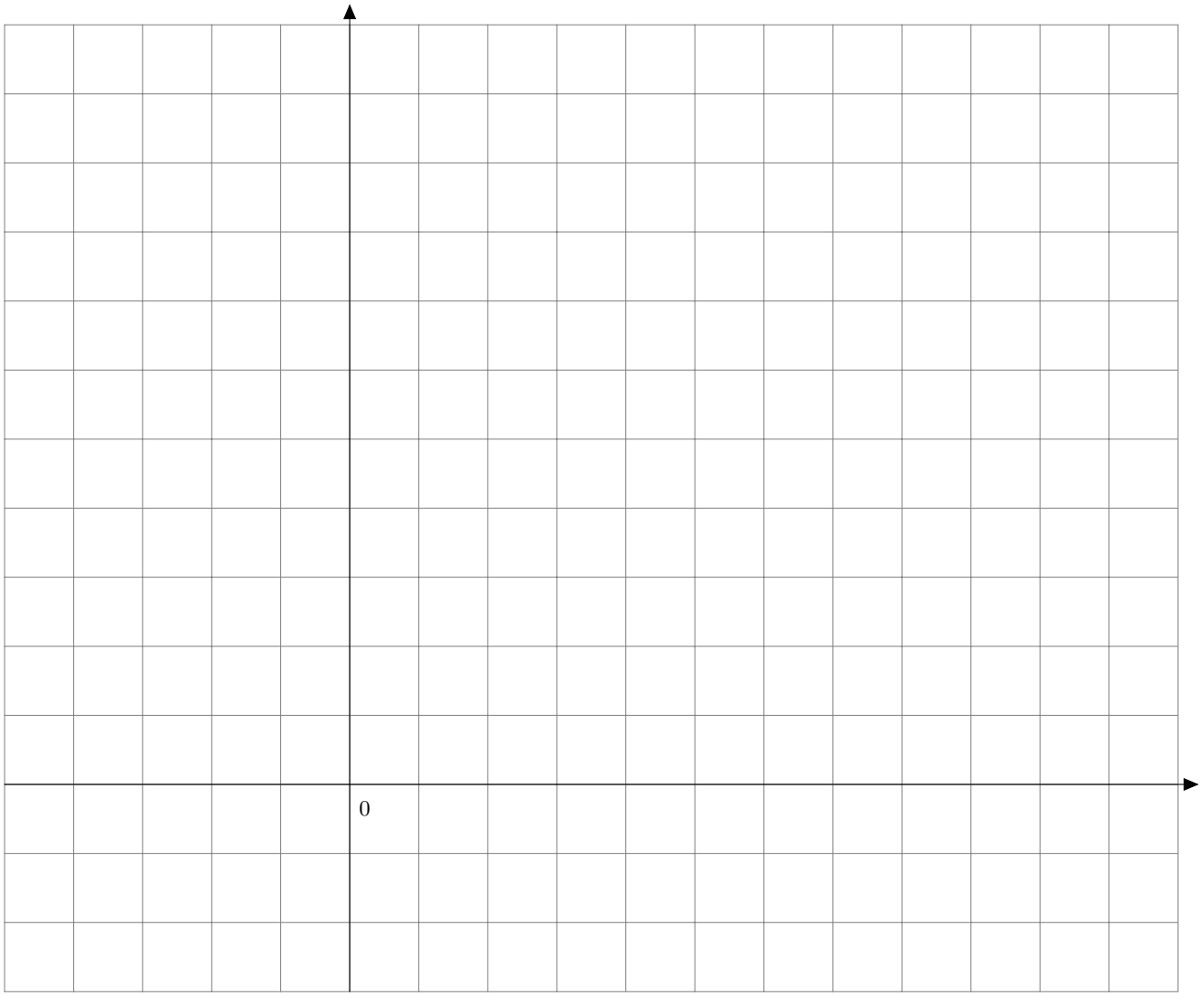
Name: _____ **No.** _____

1. A company produces two products, Product A and Product B. The production of each product requires time on two machines, Machine 1 and Machine 2. The company has a total of 100 hours of time available on Machine 1 and 200 hours of time available on Machine 2. The production of Product A requires 2 hours on Machine 1 and 4 hours on Machine 2. The production of Product B requires 3 hours on Machine 1 and 1 hour on Machine 2. The profit per unit of Product A is 10 and the profit per unit of Product B is 15. The company wants to maximize its profit.

   (a) Propose a linear programing formulation to this problem.

   (b) Use the graphical method to solve the problem.

   (c) Interpret the solution obtained.



2. A non-profit organization collects donations of medicine (M), food (F), and clothing (C). In a few days, there will be transportation to an institution, and there is a need to organize the loading. The selection team, whether selecting only medicines, clothes or food, has the capacity to process 20 tonnes. Selecting one tonne of medication takes three times longer than selecting one tonne of clothes or food. The packaging team can process 10 tonnes of donations. Packaging medication and food takes three times and twice as long as packaging clothes (in tonnes), respectively. This shipment must contain at least 2 tonnes of medication. The following LP problem was formulated to decide which goods to prepare for the next shipment:

$$
\begin{array}{lrcrcrcr}
\max & Z = 2x_M + 2x_F + x_C & & & & & & \\
\text{s.t.} & 3x_M & + & x_F & + & x_C & \leq & 20 \\
 & 3x_M & + & 2x_F & + & x_C & \leq & 10 \\
 & x_M & & & & & \geq & 2 \\
 & x_M, & & x_F, & & x_C & \geq 0 &
\end{array}
$$

Where x_j represents the tons of donation of type j ($j = M, F, C$) to prepare for shipment. The objective function translates the total utility considering that one ton of medicine has the same utility as one ton of food, which in turn is twice that of clothes. If necessary, refer to the Solver/Excel reports available in the next page to answer the following questions.

(a) Write and interpret the solution of the problem, including the value of the objective function and the slack (or auxiliary) variables.

(b) Interpret the meaning of the first slack variable.

(c) Write the dual.

(d) Determine the optimal solution of the dual (decision and slack variables) by complementarity relations.

Note: If you have not solved (2d) and need these values in the following questions, assume $y_1 = 0, y_2 = 2, y_3 = -2$.

(e) Determine the change in total utility if it were not mandatory to transport medications.

(f) What would be the new optimal solution and optimal value if the utility of a ton of medicines decreased to 1 per ton?

Microsoft Excel 16.0 Answer Report

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$E$3	max	0	8

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$9	variable xM	0	2	Contin
$C$9	variable xF	0	2	Contin
$D$9	variable xC	0	0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$5	selection capacity	8	$E$5<=$G$5	Not Binding	12
$E$6	packaging capacity	10	$E$6<=$G$6	Binding	0
$E$7	minimum meds	2	$E$7>=$G$7	Binding	0

Microsoft Excel 16.0 Sensitivity Report

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$9	variable xM	2	0	2	1	1E+30
$C$9	variable xF	2	0	2	1E+30	0
$D$9	variable xC	0	0	1	0	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$5	selection capacity	8	0	20	1E+30	12
$E$6	packaging capacity	10	1	10	24	4
$E$7	minimum meds	2	-1	2	1.333333333	2

3. Three available volunteers with the ability to perform any of three urgent tasks have been identified. The time needed to perform the tasks (T1, T2 and T3) by the volunteers (V1, V2 and V3) is shown in the following table:

	T1	T2	T3
V1	1	3	4
V2	2	1	3
V3	1	4	4

The aim is to assign a task to each of the volunteers in order to consume as little time as possible in carrying out all the tasks.

(a) Identify, among the problems studied, one that could be used to solve this problem.

(b) Propose a feasible solution to the problem and indicate the time that corresponds to it.

(c) Suppose that a fourth volunteer, V4, is identified who can only perform two of the tasks: task T2 with time 3 and task T3 with time 2. Write a formulation in ILP to include this volunteer and ensure that:

- V4 is assigned one of the tasks it can perform;
- volunteer V2 can only take on task T2 as long as V3 takes on T1.

4. A farmer must determine whether to plant corn or wheat. If he plants corn, and the weather is warm, he earns \$8 000; if he plants corn, and the weather is cold, he earns \$5 000. If he plants wheat, and the weather is warm, he earns \$7 000; if he plants wheat, and the weather is cold, he earns \$6 500. In the past 40 percent of all years have been warm and 60 percent have been cold.

(a) Build the payoff matrix.

(b) Determine Laplace action and explain its meaning.

(c) What action does Bayes' procedure recommend.

(d) Build a decision tree.

(e) Compute the expected value of perfect information.