Decision Making and Optimization

Exam Prototype 2

Exam Prototype 2 **Duration: 2h**

Notes: -Justify all answers and present the calculations carried out.
-Answer all questions using methodologies taught in Decision Making and Optimization classes.

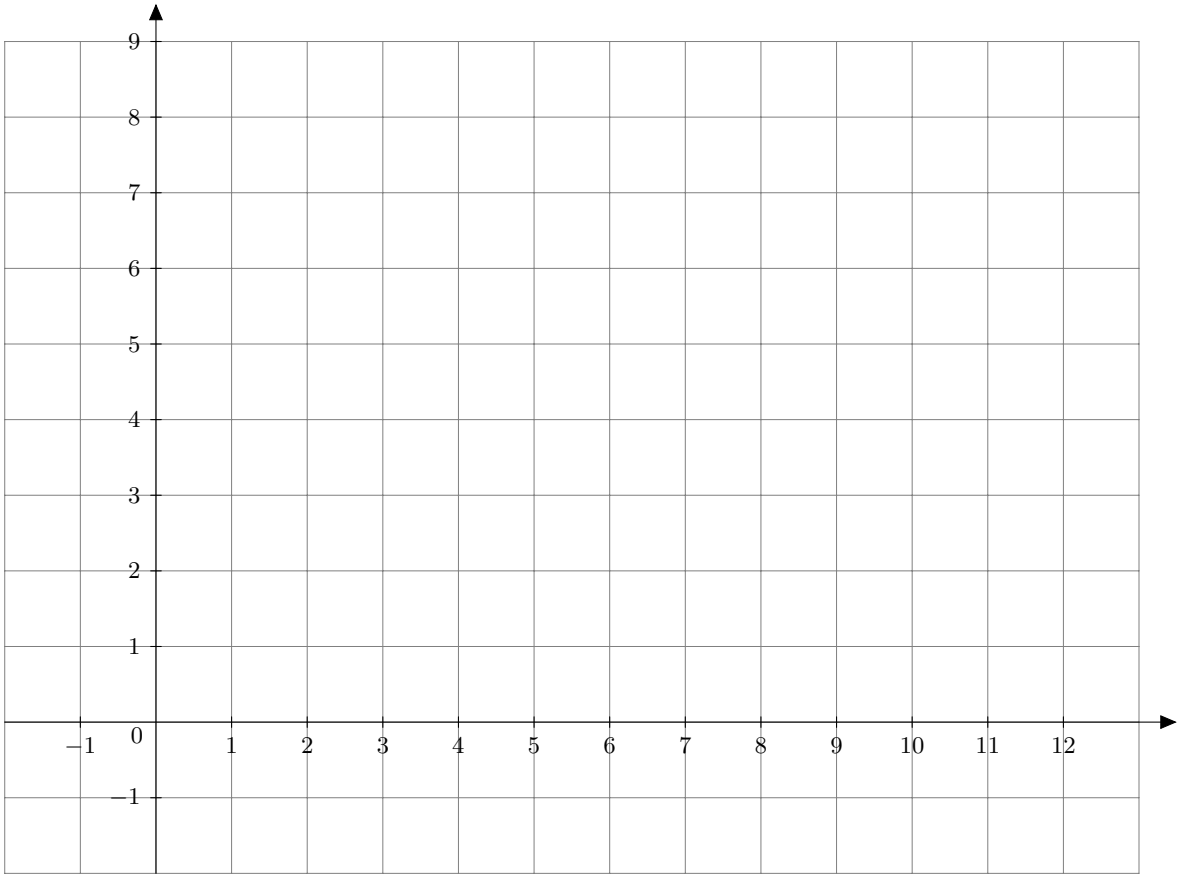
Name: _____ **No.** _____

This exam has 6 pages.
Write your answers here.
Good luck!

Question	1	2	3	4	5	6	total
Quotation	30	65	30	40	30	05	200
Classification							

1. Use the graphical method to solve the following LP problem and answer the questions that follow.

$$\begin{aligned} \max \quad & 3x_1 + 5x_2 \\ \text{s.t.} \quad & x_1 + 2x_2 \geq 4, \\ & x_1 + 3x_2 \leq 12, \\ & 3x_1 - x_2 \leq 6, \\ & x_1, x_2 \geq 0. \end{aligned}$$



(a) How many extreme points are there? What are the values of x_1 and x_2 at each one?

(b) Take four points at your choice so that you can match the corresponding solutions to the following categories:

a feasible basic solution: _____ an infeasible non-basic solution: _____

a feasible non-basic solution: _____ an infeasible basic solution: _____

(c) If the criteria is changed to minimization and the objective function is changed to $4x_1 + 2x_2$, what will the optimal solution be?

(d) Write the dual problem and the complementarity primal/dual relations.

2. C Sports produces three different types of baseball gloves: a low-cost model, a regular model and a catcher's model. The company has 900 hours of production time available in its cutting and sewing department, 300 hours available in its finishing department and 100 hours available in its packing and shipping department. Given the production time requirements and the profit contribution per glove, the following LP with decision variables x_1, x_2 and x_3 has been solved to maximize the total profit contribution.

$$
\begin{aligned}
(P)\ \max\quad & 3x_1 + 5x_2 + 8x_3 \\
\text{s.t.}\quad & \frac{1}{2}x_1 + x_2 + \frac{3}{2}x_3 \leq 900 \\
& \frac{1}{6}x_1 + \frac{1}{2}x_2 + \frac{1}{3}x_3 \leq 300 \\
& \frac{1}{8}x_1 + \frac{1}{8}x_2 + \frac{1}{4}x_3 \leq 100 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

The sensitivity report obtained using Excel Solver is shown next.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B2	x1	0	-1	3	1	1E+30
C2	x2	500	0	5	7	1
D2	x3	150	0	8	2	2

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E5	r1	725	0	900	1E+30	175
E6	r2	300	3	300	100	166,667
E7	r3	100	28	100	35	25

(a) How many gloves of each model should C Sports manufacture? Write and interpret the optimal production plan. Include the value of the slack variables.

(b) What is the total profit contribution C Sports can earn with the given production quantities?

(c) How many hours of production time will be scheduled in each department?

(d) What are the dual values for the resources? Interpret each.

(e) If overtime can be scheduled in one of the departments, where would you recommend doing so? Why?

(f) What profit contribution for low-cost gloves will lead to its production?

(g) What is the maximum amount that C Sports should agree to pay to have more hours available in its finishing department, and continue with the same production plan. How many additional hours can be considered?

(h) How much will the value of the optimal solution improve if 20 extra hours of packaging and shipping time are made available?

3. Premier Consulting has three consultants, A, B, and C, and each consultant is available for a maximum number of hours over the next four weeks, as shown below.

Consultant's Available Hours:

Consultant	A	B	C
Maximum Hours	160	160	140

The company has four clients (C1, C2, C3, C4) with projects in process. The estimated hourly requirements for each of the clients over the four-week period are as follows:

Client's Hourly Requirements:

Client	C1	C2	C3	C4
Required Hours	180	75	100	85

Hourly rates vary for the consultant–client combination and are based on several factors, including project type and the consultant's experience. The rates (euros per hour) for each consultant–client combination are as follows.

	Client C1	Client C2	Client C3	Client C4
Consultant A	100	125	115	100
Consultant B	120	135	115	120
Consultant C	155	150	140	130

The company wants to identify how many hours that each consultant should work for each client solution, with the aim of maximising the consulting firm's billings.

(a) Develop a network representation of the problem.

(b) Formulate the problem as a linear program.

(c) New information suggests that consultant A doesn't have the experience to be scheduled for client C2. If this consulting assignment is not permitted, what implications does this have for the linear program and on the total billings?

4. Brooks Development Corporation (BDC) faces the following capital budgeting decision. Six real estate projects are available for investment. The net present value and expenditures required for each project (in millions of euros) are as follows.

Project	1	2	3	4	5	6
Net Present Value (millions €)	15	5	13	14	20	9
Expenditure Required (millions €)	90	34	81	70	114	50

The budget for this investment period is 220 (millions €).

(a) Formulate a binary integer program that will enable BDC to find the projects to invest in to maximize net present value, while not exceeding the budget.

(b) Propose a lower and an upper bound for the optimal value of the problem by using the critical index. Explain how you arrived at the proposed bounds.

(c) Write the following additional conditions that limit the investment alternatives and that must be included in the binary integer program.

- At least two of projects 1, 3, 5, and 6 must be undertaken:
- If either project 3 or 5 is undertaken, they must both be undertaken:
- Project 4 cannot be undertaken unless both projects 1 and 3 also are undertaken:

5. Investment advisors have estimated stock market returns for four market segments: computers, financial, manufacturing and pharmaceuticals. The annual return projections vary depending on whether general economic conditions are improving, stable or declining. The expected annual return percentages for each market segment under each economic condition are as follows. A forecast shows

	Economic Condition		
Market Segment	**Improving**	**Stable**	**Declining**
Computers	10	2	-4
Financial	8	5	-3
Manufacturing	6	4	-2
Pharmaceuticals	6	5	-1

stable to declining economic conditions with the following probabilities: improving (0.2), stable (0.5), and declining (0.3). An individual investor wants to select one market segment for a new investment.

(a) What is the decision recommended by the Bayes criteria.

(b) Calculate the EVPI (expected value of perfect information) of the expected return percentage and on the basis of this value say what you advise.

6. Find the efficient solutions to the following multicriteria linear program, with S as shown below.

$$\begin{array}{ll} \min & x_1 - x_2 \\ \max & 2x_1 + x_2 \\ \text{s.t.} & (x_1, x_2) \in S \end{array}$$

