1 Multicriteria Optimization

1. For the following problems find the set of efficient solutions, the set of weak efficient solutions, the set of non-dominated points.

a)

$$\begin{aligned} \max \quad & z_1 = 25x_1 + 20x_2, \\ \max \quad & z_2 = x_1 + 8x_2, \\ \text{s. to:} \quad & x_1 + x_2 \leq 50, \\ & 2x_1 + x_2 \leq 80, \\ & 2x_1 + 5x_2 \leq 220, \\ & x_1, x_2 \geq 0. \end{aligned}$$

b)

$$\begin{aligned} \max \quad & z_1 = x_1, \\ \max \quad & z_2 = 3x_1 + 2x_2, \\ \text{s. to:} \quad & 1.5x_1 + 3x_2 \leq 13.5, \\ & x_1 \leq 4, \\ & 2x_1 + x_2 \leq 9, \\ & x_1, x_2 \geq 0. \end{aligned}$$

c)

$$\begin{aligned} \min \quad & z_1 = x_1, \\ \min \quad & z_2 = 3x_1 - x_2, \\ \text{s. to:} \quad & 1.5x_1 + 3x_2 \leq 13.5, \\ & x_1 \leq 4, \\ & 2x_1 + x_2 \leq 9, \\ & x_1, x_2 \geq 0. \end{aligned}$$

d)

$$\begin{aligned} \max \quad & z_1 = x_1, \\ \min \quad & z_2 = 3x_1 - x_2, \\ \text{s. to:} \quad & 1.5x_1 + 3x_2 \leq 13.5, \\ & x_1 \leq 4, \\ & 2x_1 + x_2 \leq 9, \\ & x_1, x_2 \geq 0. \end{aligned}$$

e)

$$\begin{aligned} \max \quad & z_1 = x_1, \\ \max \quad & z_2 = x_2, \\ \text{s. to:} \quad & x_2 \leq 4, \\ & x_1 + x_2 \leq 10, \\ & 2x_1 + x_2 \geq 10, \\ & x_1, x_2 \geq 0. \end{aligned}$$

f)

$$\begin{aligned} \max \quad & z_1 = 3x_1 + 2x_2, \\ \max \quad & z_2 = x_1, \\ \text{s. to:} \quad & x_1 + x_2 \leq 6, \\ & x_1 \leq 4, \\ & 2x_1 + x_2 \geq 9, \\ & x_1, x_2 \geq 0. \end{aligned}$$

g)

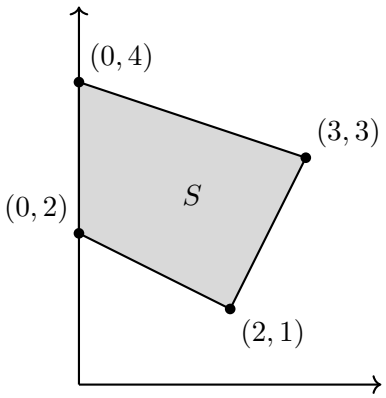
$$\begin{aligned} \min \quad & z_1 = 3x_1 + x_2, \\ \max \quad & z_2 = x_1 + x_2, \\ \text{s. to:} \quad & x_2 \leq 3, \\ & 3x_1 - x_2 \leq 6, \\ & x_1, x_2 \geq 0. \end{aligned}$$

h)

$$\begin{aligned} \max \quad & z_1 = -x_1 + 3x_2, \\ \max \quad & z_2 = 3x_1 + 3x_2, \\ \max \quad & z_3 = x_1 + 2x_2, \\ \text{s. to:} \quad & x_2 \leq 4, \\ & x_1 + 2x_2 \leq 10, \\ & 2x_1 + x_2 \leq 10, \\ & x_1, x_2 \geq 0. \end{aligned}$$

2. Find the efficient solutions to the following multicriteria linear program, with S as shown below.

$$\begin{aligned} \min \quad & x_1 - x_2 \\ \max \quad & 2x_1 + x_2 \\ \text{s.t.} \quad & (x_1, x_2) \in S \end{aligned}$$



Present all your calculations and drawings.

3. The PMat Factory manufactures two products, P1 and P2, using two departments, D1 and D2. The weekly capacity of the departments is 60 machine-hours (m.h.) and 40 labour-hours (l.h.), respectively.

   Producing one unit of P1 requires 3 m.h. in D1 and 1 l.h. in D2, and yields a profit contribution of 5 monetary units (m.u.). Producing one unit of P2 requires 2 m.h. in D1 and 2 l.h. in D2, and yields a profit contribution of 2 m.u. In addition, at least 10 units of P2 must be produced.

   The production planning process considers two distinct goals. The first is to achieve a minimum profit of 75 m.u. The second is to limit the weekly use of machine-hours in D1 to a maximum of 50 m.h., in order to reduce machine usage.

   Formulate the PMat Factory problem as a goal programming model and solve it by assigning a priority level of 2 to the profit goal and a priority level of 1 to the machine usage goal.

Some solutions

1.a) $x^* = (20, 60),\ z^* = 180,$

1.2 efficient solutions are the points $x = \alpha(0, 4) + (1 - \alpha)(3, 3)$, for $\alpha \in [0, 1]$

1.3 $(x_1, x_2, d_1^+, d_1^-, d_2^+, d_2^-) = (10, 10, 5, 0, 0, 0)$