This question is based on the paper by Robert Lucas (2003) "Macroeconomic Priorities" *AER.* in this paper he asks the question: What would be the effect on welfare if all consumption variability could be eliminated. Consider a representative household endowed with the flow of consumption:

$$C_t = Ae^{\mu t}e^{-\frac{1}{2}\sigma^2}\varepsilon_t$$

where $\ln \varepsilon_t$ is a normally distributed random variable with mean 0 and variance σ^2.

1. Show that $E(C_t) = Ae^{\mu t}$. Hint: Remember that a positive random variable Z is log-normally distributed if the natural logarithm of Z is normally distributed. Moreover, if $\ln(Z) \sim \mathcal{N}(\mu, \sigma^2)$, then $E(Z) = e^{\mu+\frac{1}{2}\sigma^2}$.

2. Assume that the lifetime utility function is

$$E\sum_{t=1}^{\infty}\beta^{t-1}\frac{1}{1-\gamma}(C_t)^{1-\gamma}$$

where β is a subjective discount rate, γ is the coefficient of risk aversion, and the expectation is taken with respect to the common distribution of the shocks ε_1, ε_2, ... A risk-averse consumer would prefer a deterministic consumption path to a risky path with the same mean. This utility difference can be quantified by multiplying the risky path by the constant factor λ in all dates and states, so that the household is indifferent between the deterministic stream and the compensated, risky stream. Formally, this amounts to solve:

$$E\sum_{t=1}^{\infty}\beta^{t-1}\frac{1}{1-\gamma}((1+\lambda)C_t)^{1-\gamma} = \sum_{t=1}^{\infty}\beta^{t-1}\frac{1}{1-\gamma}\left(Ae^{\mu t}\right)^{1-\gamma}$$

Show that the compensation parameter $\lambda \approx \frac{1}{2}\gamma\sigma^2$. Observe that according to this result the welfare gain from eliminating consumption

risk depends on the amount of risk, σ^2, and the aversion to this risk, γ.

3. To get an idea of the value to the economy as a whole of removing aggregate risk just need to compute the variance of the log of consumption about its trend, and of the coefficient of risk aversion. Using annual U.S. data for the period 1947–2001, the standard deviation of the log of real, per capita consumption about a linear trend is 0.032. Estimates of the parameter γ in use in macroeconomics and public finance applications range from 1 (log utility) to 4. Provide the interval for the compensation parameter associated with these values.

4. How valuable is optimal stabilization (monetary and fiscal) policy according to this result? What are the most critical assumptions?

5. Now consider there are no shocks in the economy and we have log utility. Consider 2 different economies: economy A and economy B. Economy A grows at rate μ_A and economy B grows at rate μ_B. Because of bad policies growth in country B is lower, i.e. $\mu_A > \mu_B$. By how much should consumers in economy B be compensated to be as well as the consumers in economy A?