1. Consider a version of our New Keynesian model. We will assume that the household's shopper receives the monetary injection **after** entering the goods market and the household's seller does not know its size when they make their pricing decision. The demand for individual i's good is given by

$$C_t(i) = D\left(\frac{P_t(i)}{\bar{P}_t}\right) = \left[\#I\frac{P(i)}{\bar{P}}\right]^{\frac{1}{\rho-1}} C_t$$

where $P_t(i)$ is his/her price, $\bar{P}_t$ is the price index, $\#I$ is the number of goods, and C_t is the composite good. Note that

$$\frac{d}{dP(i)}D\left(\frac{P(i)}{\bar{P}}\right) = D\left(\frac{P(i)}{\bar{P}}\right)\frac{1}{\rho-1}P(i)^{-1}.$$

The maximization problem is given by

$$\max_{\{C_t, P_t(i), M_{t+1}, B_{t+1}\}_{t=1,2}} \mathbb{E}\left\{\begin{array}{c} u(C_1) - v\left(D\left(\frac{P_1(i)}{\bar{P}_1}, \tau_1\right)/Z_1\right) + \beta\left[u(C_2) - v\left(D\left(\frac{P_2(i)}{\bar{P}_2}, \tau_2\right)/Z_1\right)\right] \\ +\beta^2 V(M_3, B_3) \end{array}\right\}$$

subject to

$$\begin{aligned} M_t + T_t &\geq \bar{P}_t C_t \text{ and} \\ P_t(i)D\left(\frac{P_t(i)}{\bar{P}_t}\right) + \left[M_t + T_t - \bar{P}_t C_t\right] + B_t &\geq M_{t+1} + q_t B_{t+1} \text{ for } t = 1, 2. \end{aligned}$$

where Z_t is the productivity of labor, M_t is money holdings, T_t is government transfers, B_t is public debt and q_t is the price of bonds.

A)[10pts.] Derive the first-order condition for setting the price $P_t(i)$.

B)[10pts.] Using this first-order condition, making use of the fact that $P_t(i) = \bar{P}_t$ since all prices are the same in equilibrium, and that the production function is $D\left(\frac{P_t(i)}{\bar{P}_t}, \tau_t\right) = Z_t L_t$, show that we get that

$$0 = \mathbb{E}_t\left\{-\beta^{t-1}v'(L_t)L_t + \mu_t \rho Z_t L_t \bar{P}_t\right\},$$

where μ_t is the Lagrange multiplier of the budget constraint.

C)[10pts.] Derive the first-order conditions for consumption and money. Remember that these decisions are taken after all uncertainty is resolved and hence we get our standard conditions.

D)[10pts.] Assume $u(c) = \log(C)$ and the cash-in-advance constraint holds, so

$$\bar{P}_t = \frac{M_t(1+\tau_t)}{Z_t L_t},$$

Show that you can use all this to derive the following

$$0 = \mathbb{E}_t\left\{-v'(L_t)L_t + \beta\frac{1}{(1+\tau_{t+1})}\rho\right\} \quad (1)$$

E)[10pts.] To finish deriving an expectational Phillips curve (a relationship between employment and inflation), remember that the realized level of labor, L_t bears the following relationship to the target level of labor $\bar{L}_t$,

$$L_t = \frac{(1+\tau_t)}{(1+\bar{\tau})}\bar{L}_t. \tag{2}$$

where $\bar{\tau}$ is the expected level of money growth. Assume $v(L) = \frac{L^{1+\gamma}}{1+\gamma}$. If we plug the target level of labor into equation (1), we get an expression for how this target will respond to the expected growth rate of money, as an approximation this can be taken to be

$$\bar{L}_t^{1+\gamma} = \frac{\beta\rho}{1+\bar{\tau}_t} \tag{3}$$

Consider a country that is running a hyperinflation, so $\bar{\tau}$ is very large. This country then slows the growth rate of money abruptly. Discuss what you would expect to have occur under the following scenarios:

1. Expectations gradually adjust down to the new low level.
2. Expectations adjust immediately.

Please couch your discussion in terms of conditions, (1) and (2).