Universidade de Lisboa
Instituto Superior de Economia e Gestão
PhD in Economics
Advanced Macroconomics
Part I
Lecturer: Bernardino Adão
Year: 2021-2022
Exam: **First Exam**
Date: 17.1.2022
Schedule: 18:00-21:00
Instructions:
• This is an open book exam.
• The use of electronic devices is forbidden.
• The answers to each part of the exam should be provided in a separate set of pages.
• Answers should be concise. Any answer not strictly related to the question may receive a negative evaluation.
• The two questions of Part I have multiple subquestions and they require some cleverness, so be careful. The subquestions build on each other, so it is important to take your time and get a subquestion right before moving on to the next subquestion.

Part I

[6 pts] 1. Consider a version of our model that incorporates a direct cost to holding money, you can lose it. We will implicitly assume that everyone enters into an insurance pool. With this assumption, the amount that a person loses is just a deterministic fraction of their total money holdings. Let δ denote the fraction of your money that you lose. Then if M_{t+1} is the amount of money you show up with next period, you must have actually set aside $M_{t+1}/(1-\delta)$. In this case the household's problem becomes

$$\max_{\{C_t, L_t, M_{t+1}, B_{t+1}\}_{t=1,2}} u(C_1) - v(L_1) + \beta\left[u(C_2) - v(L_2)\right] + \beta^2 V(M_3, B_3) \text{ subject to}$$

$$\begin{aligned} M_t &\geq P_t C_t \text{ and} \\ P_t Z_t L_t + [M_t - P_t C_t] + B_t + T_t &\geq M_{t+1}/(1-\delta) + q_t B_{t+1} \text{ for } t = 1, 2. \end{aligned}$$

Let $\bar{M}_t$ be the amount of money that they have at the beginning of the period, then $T_t = \tau \bar{M}_t$ is the transfer that they receive in the asset market. Thus, the total money stock in the asset market is $\bar{M}_t(1+\tau)$, but then between periods, the fraction δ is lost and disappears, so $\bar{M}_{t+1}$ is less than this amount.

A) [1 pt.] Write down the Lagrangian for this problem and determine the first-order conditions for the optimal levels of consumption, labor, money and bonds.

B) [1 pt.] Your first-order condition for money should be different (from the normal condition) to account for our cost of holding it between periods. Explain why the presence of this cost can allow for a negative interest rate, which here means $q_t > 1$.

C) [1 pt.] How will you need to adjust the updating condition for the money supply $\bar{M}_t$? Assume that we have a steady state in which labor is constant at $L_t = L$ and that productivity grows at constant rate $Z_{t+1} = (1+g)Z_t$. If the cash-in-advance constraint holds and the goods market clearing condition implies consumption is equal to output, what does this say about the growth rate of prices P_{t+1}/P_t?

D) [1 pt.] Assume that we have log preferences over consumption so $u(C_t) = \log(C_t)$ and a power function for labor $v(L_t) = \dfrac{L_t^{1+\gamma}}{1+\gamma}$. This will allow us to have a steady state. Given this assumption, try and construct the change in variables that you will need to make in order to determine the equilibrium level of steady state labor. Start by using the labor condition to determine the change to μ_t and then given that, use the consumption condition to show that the same change works for λ_t.

E. [1 pt.] Try to boil down our equilibrium conditions into a 3-equation system in L, $\tilde{\mu}$ and $\tilde{\lambda}$ and discuss how you could solve this to determine an equilibrium for our model. (Don't write code, just chat about how this should be done.)

F. [1 pt.] Fixing the growth rate in prices, speculate on how a change in the loss factor δ with an offsetting change τ would affect the steady state level of labor? Can this model deliver different steady state levels of labor for the same rate of inflation?

[4 pts] 2. A major tax that we have not considered is a consumption tax. Consumption taxes are those paid by consumers when they purchase goods. So, instead of paying the price per unit, p, they pay $(1+\tau_c)p$. Then the seller keeps p while the amount $\tau_c p$ is turned over to the government. We are going to try and extend our basic model to allow for these kinds of taxes. To keep things simple we will drop labor and capital taxes. As a result, we can go back to the original backyard model of production in which every household simply produces output using their own labor and capital.

Now we want to augment our basic model to include consumption taxes. Here is a basic layout of the model. The equation of motion for the capital stock is

$$K_{t+1} = (1-\delta)K_t + X_t.$$

The resource constraint for the economy is given by

$$[Z_t L_t]^{1-\alpha} K_t^{\alpha} = C_t + X_t.$$

Assume that both productivity and the money supply grow at constant rates given by

$$Z_t = (1+g)Z_{t-1}, \text{ and } M_t = (1+\tau)M_{t-1}.$$

respectively. Make the normalization that $Z_1 = 1$, then $Z_t = (1+g)^{t-1}$, so it is just the accumulated growth factor. This will turn out to be notationally convenient.

We can write the household's choice problem as choosing a sequence of quantities $\{C_t, L_t, M_{t+1}, B_{t+1}, K_{t+1}\}_{t=1}^{2}$ so as to maximize

$$\max \sum_{t=1,2} \beta^{t-1}\left[u(C_t) - v(L_t)\right] + \beta^2 V(M_3, B_3, K_3)$$

subject to

$$M_t \geq (1+\tau_c)P_t C_t \text{ and}$$

$$\begin{aligned} & P_t\left[[Z_t L_t]^{1-\alpha} K_t^{\alpha} - \delta K_t\right] + [M_t - (1+\tau_c)P_t C_t] + B_{t-1} + T_t \\ \geq \quad & M_{t+1} + q_t B_{t+1} + P_t\left[K_{t+1} - K_t\right] \text{ for all } t \leq 2. \end{aligned}$$

We will assume that the revenue collected from the consumption taxes are lump-sum rebated to the households. The key assumption here is that you receive the per capita level collected as a transfer along with the money injection. So, your transfers are independent of your individual tax payments.

A) [1 pt.] Set-up the Lagrangian and derive the first-order conditions.

B) [1 pt.] Assume that we have a balanced growth path in which: $L_t = L$, i.e., labor is constant; and the marginal product of capital is constant so $K_t = Z_t K$. Given this, show that (i) $Y_t = Z_t Y$, (ii) $X_t = Z_t X = Z_t K_t (g+\delta)$, and hence that (iii) $C_t = Z_t C$. Finally assume that the cash-in-advance constraint binds and use that to pin down the price level.

C) [1 pt.] Plug these results into your first-order conditions, and come up with a change-in-variables to render things stationary. Use this change to generate some nice stationary conditions which we can use to solve out for the balanced growth path.

D) [1 pt.] How do consumption taxes differ from labor and capital taxes? In the context of a representative agent model, which form of taxation is likely to be superior?