1. Consider a version of our New Keynesian model. We will assume that the household's shopper receives the monetary injection **after** entering the goods market and the household's seller does not know its size when they make their pricing decision. The demand for individual i's good is given by

$$C_t(i) = D\left(\frac{P_t(i)}{\bar{P}_t}\right) = \left[\#I\frac{P(i)}{\bar{P}}\right]^{\frac{1}{\rho-1}} C_t$$

where $P_t(i)$ is his/her price, $\bar{P}_t$ is the price index, $\#I$ is the number of goods, and C_t is the composite good. Note that

$$\frac{d}{dP(i)}D\left(\frac{P(i)}{\bar{P}}\right) = D\left(\frac{P(i)}{\bar{P}}\right)\frac{1}{\rho-1}P(i)^{-1}.$$

The maximization problem is given by

$$\max_{\{C_t,P_t(i),M_{t+1},B_{t+1}\}_{t=1,2}} \mathbb{E}\left\{\begin{array}{c} u(C_1) - v\left(D\left(\frac{P_1(i)}{\bar{P}_1},\tau_1\right)/Z_1\right) + \beta\left[u(C_2) - v\left(D\left(\frac{P_2(i)}{\bar{P}_2},\tau_2\right)/Z_1\right)\right] \\ +\beta^2 V(M_3, B_3) \end{array}\right\}$$

subject to

$$\begin{aligned} M_t + T_t &\geq \bar{P}_t C_t \text{ and} \\ P_t(i)D\left(\frac{P_t(i)}{\bar{P}_t}\right) + \left[M_t + T_t - \bar{P}_t C_t\right] + B_t &\geq M_{t+1} + q_t B_{t+1} \text{ for } t = 1, 2. \end{aligned}$$

where Z_t is the productivity of labor, M_t is money holdings, T_t is government transfers, B_t is public debt and q_t is the price of bonds.

A) Derive the first-order condition for setting the price $P_t(i)$.

B) Using this first-order condition, making use of the fact that $P_t(i) = \bar{P}_t$ since all prices are the same in equilibrium, and that the production function is $D\left(\frac{P_t(i)}{\bar{P}_t},\tau_t\right) = Z_t L_t$, show that we get that

$$0 = \mathbb{E}_t\left\{-\beta^{t-1}v'(L_t)L_t + \mu_t \rho Z_t L_t \bar{P}_t\right\},$$

where μ_t is the Lagrange multiplier of the budget constraint.

C) Derive the first-order conditions for consumption and money. Remember that these decisions are taken after all uncertainty is resolved and hence we get our standard conditions.