Foundations of Financial Economics
Bernardino Adao
Final Exam
June, 30, 2023
Total time: 2 hours. Total points: 20

Instructions:

Please sit in alternate seats. This is a closed-book, closed-note exam. Please get rid of everything but pen/pencil. In your answer explain all the steps in your reasoning. Keep answers short; I don't give more credit for long answers, and I can take points off if you add things that are wrong or irrelevant.

Formulas:

If x and y are random variables then

$$E(xy) = ExEy + cov(x, y) \, ; \, var(x) = Ex^2 - (Ex)^2 \, ;$$

$$var(y + x) = var(y) + var(x) + 2cov(y, x);$$

$$cov(kx, y) = kcov(x, y) \, ; \, var(kx) = k^2 var(x)$$

If x is normal distributed then $\exp(x)$ is lognormal, and

$$E \exp(x) = \exp(Ex + 0.5\sigma^2(x))$$

Brownian motion

$$z_{t+\Delta} - z_t \sim N(0, \Delta)$$

Differential

$$dz_t = lim_{\Delta \searrow 0}(z_{t+\Delta} - z_t)$$

$$dz_t^2 = dt, \; dz_t dt = 0, \; dt^\alpha = 0, \text{ if } \alpha > 1$$

$$E_t(dz_t) = 0, var_t(dz_t) = E_t(dz_t^2) = dz_t^2 = dt$$

Ito's Lemma

$$df(x, t) = f_t dt + f_x dx + \frac{1}{2} f_{xx} dx^2 = \left(f_t + f_x \mu_x + \frac{1}{2} f_{xx} \sigma_x^2 \right) dt + f_x \sigma_x dz$$

Questions:

1. This group of questions concerns basic concepts.

(1 pt) a. Explain how the price of a stock can be computed.

(1 pt) b. What are the 3 Fama-French factors?

(1 pt) c. What is the equity premium?

(1 pt) d. Is the equity premium predictable? Explain.

(1 pt) e. What is the relationship between the risk free return rate and the risky return rate?

(1 pt) f. What is the intertemporal marginal rate of substitution? And the risk aversion? Are they related?

(1 pt) g. What are the Hansen-Jagannathan bounds?

(1 pt) h. Prove that the volatility of the return of a portfolio decreases with the covariance between the returns of the securities that integrate the portfolio.

(1 pt) i. Explain how financial markets help investors decrease idiosyncratic risk.

(1 pt) j. How is the riskiness of a security measured in the CAPM?

(1 pt) k. What is a geometric Brownian motion? Give an example of its use in financial economics.

2. Consider a representative agent economy. The consumer maximizes $E_0 \sum_{t=0}^{\infty} \beta^t u(C_t)$. The economy has a firm. The stock of the firm pays dividend $\mathcal{D}_t$ and the price of the stock is $\mathcal{S}_t$. The consumers issue zero coupon bonds. Let $\mathcal{B}_{j,t}$ be the price at t of the zero coupon bond that matures at $t+j$. Because they are zero coupon bonds at maturity, $\mathcal{B}_{0,t} = 1$. There are 2 maturities of debt. Denote by $B_{j,t}$ the household's holdings of a bond with maturity j, and denote by S_t the holdings of the shares. These quantities are chosen by the household in period $t-1$. A bond with maturity j in period $t-1$ has maturity $j-1$ in period t. The budget constraint of the representative consumer is

$$B_{1,t+1}\mathcal{B}_{1,t} + B_{2,t+1}\mathcal{B}_{2,t} + S_{t+1}\mathcal{S}_t + C_t = B_{1,t}\mathcal{B}_{0,t} + B_{2,t}\mathcal{B}_{1,t} + S_t\left(\mathcal{D}_t + \mathcal{S}_t\right)$$

(1 pt) a. What are the first order conditions of the consumer's problem?

(1 pt) b. Obtain from the first order conditions the equation for the short-term (one-period) risk-free interest rate. $\left(R_{1,t}^f\right)^{-1} = E_t\left[\frac{\beta u'(C_{t+1})}{u'(C_t)}\right]$.

(1 pt) c. Let $R_{2,t}^f$ be the risk-free real gross return between periods t and $t+2$. Show that $\left(R_{2,t}^f\right)^{-1} = E_t\left\{\frac{\beta^j u'(C_{t+2})}{u'(C_t)}\right\}$.

(1 pt) d. Let $\Delta c_{t+1} \equiv \ln C_{t+1} - \ln C_t$. Assume that Δc_{t+1} is an i.i.d. normal distribution with mean μ and variance σ^2. Let the preferences be $u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$. Obtain the analytical solution for the short-term (one-period) interest rate.

(1 pt) e. Is the interest rate positively correlated with consumption growth?

(1 pt) f. Define the long-term (two-period) interest rate as the square root of $R_{2,t}^f$. Show that the long-term (two-period) interest rate satisfies the equation:

$$\frac{1}{R_{2,t}} = \frac{1}{R_{1,t}} E_t\left[\frac{1}{R_{1,t+1}}\right] + cov_t\left[\frac{\beta u'(C_{t+1})}{u'(C_t)}, \frac{\beta u'(C_{t+2})}{u'(C_{t+1})}\right].$$

(1 pt) g. Is the long-term interest rate the average of the expected short-term interest rates? Does this depend on the fact that Δc_{t+1} is i.i.d. ? What if $\gamma = 0$?

(1 pt) h. Consider now that consumption growth follows the process, $\Delta c_{t+1} = \alpha s_t + \varepsilon_{t+1}$, $\alpha > 0$, where s_t is an i.i.d. process, independent from ε_{t+1}, that can take only two values, $\{0, 1\}$, with equal probability. All variables with subscript t are known at time t. The ε_{t+1} is an i.i.d. normal distribution with mean μ_ε and variance σ_ε^2. Obtain the analytical solution for the long-term interest rate.

(1 pt) i. Can the long-term rate be smaller than the short-term rate? Why?