3 Transportation Model and Variants

1. MG Auto has three manufacturing plants in Los Angeles, Detroit, and New Orleans, as well as two major distribution centers in Denver and Miami. The three plants have quarterly capacities of 1000, 1500, and 1200 cars, respectively, while the two distribution centers have quarterly demands of 2300 and 1400 cars, respectively. The mileage chart showing the distance between the plants and the distribution centers is given in the table below.

	Distribution Centers		
Plants	Denver	Miami	Capacity
Los Angeles	1027	2342	1000
Detroit	1158	1086	1500
New Orleans	1303	661	1200
Demand	2300	1400	3700

The trucking company in charge of transporting the cars charges 8 cents per mile per car. Model the problem, and use a solver to find the transportation plan that minimizes the total cost.

2. Three refineries (R1, R2 and R3) with daily production capacities of 25 000, 15 000 and 5 000 tons of gas, respectively, supply three large distribution centers (D1, D2 and D3) whose daily demands are 15 000, 10 000 and 20 000 tons. Gas is transported through a pipeline network at a cost of 200 m.u. per ton per km. The distances (in km) between refineries and distribution centers are given in the following table:

	D1	**D2**	**D3**
R1	5	70	320
R2	75	15	220
R3	300	200	2

The objective is to determine the optimal transportation plan that satisfies all demands at minimum cost while ensuring high service quality.

   (a) Formulate the problem as a linear programming problem.

   (b) Use a solver to obtain the optimal solution.

   (c) Modify and re-solve the model assuming refinery R2 has stopped producing gas.

   (d) Modify and re-solve the model assuming the demand at distribution center D3 has decreased to 10 000 tons.

   (e) Modify and re-solve the model assuming the production capacity of refinery R1 is reduced to 20 000 tons.

3. A company produces a certain product in two factories (F1 and F2) and distributes it to three selling points (S1, S2 and S3). The maximum production capacities for the upcoming period are 400 tons at factory F1 and 800 tons at factory F2. The potential sales volumes at the three selling points are 400, 500, and 300 tons, respectively. Transportation costs (in hundreds of monetary units per ton) between each factory and each selling point are shown in the table below:

	S1	**S2**	**S3**
F1	10	20	25
F2	25	15	20

The objective of the company's management is to maximize total profit, defined as total revenue minus total transportation costs. Model the problem to determine the optimal distribution plan and the corresponding maximum profit.

4. The METRO WATER DISTRICT is the agency responsible for managing water distribution across a large geographic region. Its main customers are four cities—Berdoo (C1), Los Devils (C2), Sam Go (C3), and Hollyglass (C4). Water is supplied from three rivers: Colombo (R1), Sacron (R2), and Calorie (R3). Each city can be supplied from any river, except city C4, which cannot receive water from river R3.

   The table below presents the cost (in monetary units) of transporting one million kiloliters (Kl) of water from river R_i to city C_j, along with the water availabilities at each river and the minimum/maximum needs of each city.

	C1	**C2**	**C3**	**C4**	Availabilities (millions of Kl)
R1	16	13	22	17	50
R2	14	13	19	15	60
R3	19	20	23	×	50
Minimum needed (millions of Kl)	30	70	0	10	
Requested (millions of Kl)	50	70	30	∞	

   Management wants to distribute all available water from the three rivers to the four cities in such a way that:

   (a) All essential minimum needs are satisfied.

   (b) Requested amounts may be exceeded up to their upper limits (where applicable).

   (c) The special constraint (C4 cannot receive water from R3) is respected.

   (d) Total transportation cost is minimized.

   Model the problem and use a solver to determine the optimal allocation of water from the rivers to the cities.

5. The company BETTER PRODUCTS plans to launch four new products (P1, P2, P3 and P4), using three plants (F1, F2 and F3) that currently have excess production capacity. Because the production effort per unit is comparable across products, each plant's available capacity is measured in terms of the number of units of any product it can produce per day, as shown in the table below. The bottom row indicates the required daily production levels needed to meet projected demand.

   Each plant can produce any of the four products, except that plant F2 cannot produce product P3. However, the variable production cost per unit varies by plant and by product, as shown in the table.

Unit production cost (m.u.)	**P1**	**P2**	**P3**	**P4**	Daily capacity (units)
F1	41	27	28	24	75
F2	40	20	×	23	75
F3	37	30	27	21	45
Required daily production (units)	20	50	30	50	

   Management must now determine how to allocate production of the four products across the three plants. Two policy options are being considered:

   (a) Allow product splitting, meaning a given product may be produced in more than one plant.

   (b) Prohibit product splitting, meaning each product must be produced entirely in a single plant (i.e., a plant can produce at most one product).

   Write a report to support management in choosing between these two options, including a clear model formulation and an evaluation of the optimal production plan under each policy.

6. A department has opened three vacancies for translators:

   Vacancy 1: Portuguese/French;

   Vacancy 2: Portuguese/German;

   Vacancy 3: Portuguese/Greek.

   Four candidates applied and in the selection tests they achieved the following grades (in scale from a minimum of zero to a maximum of ten):

Candidate	Portuguese/French	Portuguese/German	Portuguese/Greek
A	8.5	7.0	6.0
B	7.5	8.0	6.5
C	6.0	7.5	8.5
D	7.0	6.5	8.0

   Determine the assignment that provides the best service quality.

Some solutions

3.1 $x^* = (1000, 0, 1300, 200, 0, 1200),\ z^* = 313200.$

3.3 $x^* = (400, 0, 0, 0, 500, 300),\ z^* = 3500.$

3.4 $x^* = (0, 50, 0, 0, 0, 20, 0, 40, 50, 0, 0, 0),\ z^* = 2460.$

3.5(a) $x^* = (0, 0, 30, 0, 0, 50, 0, 25, 20, 0, 0, 25),\ z^* = 3680.$

3.5(b) F1 produces P3, F2 produces P2, F3 produces P1, $x^* = (0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0),\ z^* = 2580.$

3.6 V1 for C3, V2 for C2, V3 for C4, $x^* = (0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1),\ z^* = 22.$