1. A company can produce three products (P_1, P_2, and P_3) with unit contribution margins of 12, 14, and 11 monetary units, respectively. In order to determine the quantities of each product to be produced so as to maximize the total contribution margin, taking into account the company's operating conditions (120 man-hours and 140 machine-hours available) and the characteristics of the market in which it operates, the production manager formulated and solved the following model:

$$
\begin{array}{lll}
(P) \quad \max & z = 12x_1 + 14x_2 + 11x_3 & \\
\text{s.t.} & 2x_1 + 2x_2 + x_3 & \leq 120 \\
& x_1 + x_2 + 2x_3 & \leq 140 \\
& x_1 + x_2 & \leq 60 \\
& x_3 & \geq 48 \\
& x_1,\, x_2,\, x_3 & \geq 0
\end{array}
$$

The final simplex table is:

BV	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
z	2	0	0	$\frac{17}{3}$	$\frac{8}{3}$	0	0	$\frac{3160}{3}$
x_2	1	1	0	$\frac{2}{3}$	$-\frac{1}{3}$	0	0	$\frac{100}{3}$
x_7	0	0	0	$-\frac{1}{3}$	$\frac{2}{3}$	0	1	$\frac{16}{3}$
x_6	0	0	0	$-\frac{2}{3}$	$\frac{1}{3}$	1	0	$\frac{80}{3}$
x_3	0	0	1	$-\frac{1}{3}$	$\frac{2}{3}$	0	0	$\frac{160}{3}$

a) Prepare a report to be presented to the company's manager, including the data and comments that you consider relevant for decision-making.

b) A new tax on polluting products will be introduced, requiring the payment of α monetary units for each unit produced, with no possibility of passing any effect on to consumers. Knowing that, among the three products, only product P_2 will be subject to the new tax, study the consequences for the company of the entry into force of this tax and prepare a report to be presented to the manager.

c) Based on a new market study, the manager recommended that the production of product P_2 should be at least twice the production of product P_3. Analyze this recommendation and prepare a report with the conclusions of the analysis.

2. A public-sector industrial company intends to determine the monthly production levels of three of the main products in its range. These products are highly polluting and have generated significant opposition from populations living near the company's production facilities. Taking into account its social responsibility, the management accepted the following Linear Programming model as appropriate:

$$
\begin{aligned}
(P) \quad \min \quad & z = 3x_1 + x_2 + 2x_3 \\
\text{s.t.} \quad & x_1 + x_2 + x_3 \leq 80 \\
& 2x_1 + x_2 + 1.25x_3 \geq 100 \\
& x_1,\, x_2,\, x_3 \geq 0
\end{aligned}
$$

The optimal tableau associated with the problem is the following:

B.V.	x_1	x_2	x_3	x_4	x_5	RHS
z	0	0	$-\frac{1}{2}$	-1	-2	120
x_2	0	1	$\frac{3}{4}$	2	1	60
x_1	1	0	$\frac{1}{4}$	-1	-1	20

This model aims to minimize the level of pollution generated by the products, while satisfying two essential constraints: a total production volume of at most 80 thousand tons, and a total gross margin of at least 100 thousand euros, in order to cover monthly fixed costs amounting to 95 thousand euros. The pollution level is measured in tons of gas released into the atmosphere per thousand tons of production.

a) Write the optimal values of the decision variables and the auxiliary variables of the primal and the dual problems, and interpret their economic meaning.

b) The manager of the production sector argued before the management that increasing the overall productive capacity would allow a significant reduction in pollution. In turn, the financial manager estimated that a 30% increase in capacity would imply an increase in monthly fixed costs of 10 thousand euros. The management accepts the proposal provided that the monthly profit is maintained. Determine the best solution for the company.

c) Protests by environmental groups have been intensifying, and the administration's arguments that the company's objective has been to minimize the volume of pollution have proven insufficient. Given that product 1 has been heavily criticized for being highly polluting, the management decided to reduce its production by at least 50%. Study the consequences of this decision and assess whether or not it is a sound decision.

3. A company in the food sector produces four products monthly: a gourmet product, sold at 28 monetary units per package; two mid-range products, sold at 16 and 12 monetary units per package; and a more economical product, sold at 10 monetary units per package. To determine the optimal monthly production plan, the following model was constructed:

$$
\begin{aligned}
\max \quad & z = 28x_1 + 16x_2 + 12x_3 + 10x_4 \\
\text{s.t.} \quad & 4x_1 + 6x_2 + 4x_3 + 2x_4 \leq 576 && \text{[Labor]} \\
& 10x_1 + 8x_2 + 12x_3 + 16x_4 \leq 1600 && \text{[Energy]} \\
& x_2 + x_3 \geq 60 && \text{[Market]} \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}
$$

The optimal Simplex tableau is the following:

B.V.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
z (max)	0.00	10.00	0.00	4.00	7.00	0.00	16.00	3072.00
x_1	1.00	0.50	0.00	0.50	0.25	0.00	1.00	84.00
x_6	0.00	−9.00	0.00	11.00	−2.50	1.00	2.00	40.00
x_3	0.00	1.00	1.00	0.00	0.00	0.00	−1.00	60.00

a) Prepare a report on the relevant aspects of the production plan corresponding to the solution of the model, to be presented to the company management.

b) Analyze the sensitivity of the solution with respect to the selling prices of the mid-range products and comment on the results obtained.

c) Provide a suggestion for a modification regarding the most economical product that would ensure its inclusion in the company's production plan. Justify your choices.

d) If it were decided that the production of these four products had to reach at least 145 packages, how should the production manager proceed? What would be the consequence for sales revenue?

Tabela 1: Answer Report.

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$D$3	Max	5805	5805

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$2	x1	0,45	0,45	Contin
$C$2	x2	0,84	0,84	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$5	orçamento	5610	$D$5<=$F$5	Not Binding	390
$D$6	horas	600	$D$6<=$F$6	Binding	0
$D$7	min P1	0,45	$D$7>=$F$7	Not Binding	0,25
$D$8	max P1	0,45	$D$8<=$F$8	Binding	0
$D$9	min P2	0,84	$D$9>=$F$9	Not Binding	0,59
$D$10	max P2	0,84	$D$10<=$F$10	Not Binding	0,16

Tabela 2: Sensitivity Report.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$2	x1	0,45	0	4500	1E+30	900
$C$2	x2	0,84	0	4500	1125	4500

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$5	orçamento	5610	0	6000	1E+30	390
$D$6	horas	600	9	600	48,75	295
$D$7	min P1	0,45	0	0,2	0,25	1E+30
$D$8	max P1	0,45	900	0,45	0,216666667	0,2
$D$9	min P2	0,84	0	0,25	0,59	1E+30
$D$10	max P2	0,84	0	1	1E+30	0,16

4. A photography and cinema student won 10,000 euros in a photojournalism contest. He reserved 4,000 euros to pay for an underwater photography course and intends to invest the remaining amount in projects to gain professional experience and generate income for a postgraduate program.

   After analyzing several alternatives, he selected two projects that seemed interesting. In the first project, the partner needs to have a participation between 20% and 45%. In the second project, the partner needs to have a participation of at least 25%. The first project requires 5,000 euros and 400 hours of work. The second project requires only 4,000 euros but demands more working hours: 500 instead of 400. Each of the projects expects to generate a profit of 4,500 euros. The student would like to participate in both projects provided that he does not have to work more than 600 hours and manages to earn at least 5,000 euros.

   Before making a decision, he decided to consult a friend. In response, the friend sent him a file with information about the projects, but the student is not able to understand it. The information the friend sent is in the Tables. 1 and 2.

   Write a text explaining the information sent by the friend clearly and in detail, to help the student make the investment decision in the most informed way possible.

5. A company that can produce 4 products formulated and solved the following problem to determine the quantity to produce of each product:

$$\begin{aligned} \max \quad & z = 10x_1 + 12x_2 + 8x_3 + 20x_4 \\ \text{s.t.} \quad & 2x_1 + 4x_2 + 2x_3 + 6x_4 \leq 124 \\ & x_1 + x_2 \geq 30 \\ & x_3 + x_4 \geq 24 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

   The first constraint represents the limitation on the energy that can be consumed in the production of the 4 products, and the other two were suggested by the marketing department to ensure visibility in the 2 market segments in which the company operates. The optimal table is the following:

B.V.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
z (max)	0	8	0	8	5	0	2	572
x_6	0	1	0	2	½	1	1	8
x_1	1	2	0	2	½	0	1	38
x_3	0	0	1	1	0	0	-1	24

   a) Prepare a report for the marketing department to support the re-evaluation that the department intends to make regarding the conditions it suggested.

   b) It is feared that in the near future it may be necessary to reduce energy consumption, with the expectation that later the current level could be restored or even exceeded. Conduct a study that allows you to answer the following questions:

      - **Q1.** What is the threshold for energy consumption below which the company will no longer be able to follow the current recommendations of the marketing department?

- **Q2.** When the situation improves, will it be worth exceeding the current level? By how much? With what consequences?

6. A company intends to determine the monthly production quantity of 4 products (P_1, P_2, P_3, and P_4) which are expected to be sold at unit prices of 3, 5, 4, and 6 monetary units, respectively. The company plans to allocate, at most, 270 tons of raw material and 480 hours of labor per month to the production of these products, and it considers it necessary to achieve a minimum monthly production of 120 units.

   The company's technicians built a Linear Programming (LP) model whose solution resulted in the following tables:

B.V.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
z^a (min)	1.0	1.0	1.0	1.0	0.0	0.0	-1.0	120.0
x_5	2.0	3.0	5.0	1.0	1.0	0.0	0.0	270.0
x_6	4.0	4.0	2.0	6.0	0.0	1.0	0.0	480.0
R_1	1.0	1.0	1.0	1.0	0.0	0.0	-1.0	120.0

Phase 1 - initial table

B.V.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
z (max)	0.0	0.0	0.0	0.0	2.0	2.5	11.0	420.0
x_4	0.0	0.5	0.0	1.0	0.5	0.75	4.0	15.0
x_1	1.0	0.0	0.0	0.0	-1.0	-1.0	-7.0	90.0
x_3	0.0	0.5	1.0	0.0	0.5	0.25	2.0	15.0

Phase 2 - final table

   a) Would it be advantageous for the company to allocate an additional 70 hours of labor per month to the production of these products? If so, under what conditions?

   b) One of the partners believes that the minimum monthly production of these products should be reduced by 5%, while the other partner believes it should be reduced by 10%. Study both options and prepare a concise report with the conclusions reached.

   c) Perform a sensitivity analysis on the selling prices of the two most expensive products. Comment on the results obtained.

   d) One partner argues that the monthly production of the two cheapest products should be limited to 100 units, while the other partner disagrees. Conduct a study that allows each partner to justify their respective position.

7. A footwear manufacturer is planning next year's collection. This collection includes three types of boots: *Walker*, *Hicker*, and *Backpacker*. Among the resources used in production, three are particularly expensive:

- **NOwater** – a coating that makes the boots waterproof,
- **Fabrínsula** – an insulator to retain heat,
- **Lugster** – soles.

The following table shows the resources required to produce each type of boot:

Resource	Walker	Hicker	Backpacker
NOwater [sq.ft. per pair]	1/2	1	5/2
Fabrínsula [oz. per pair]	1/2	2/3	4/3
Lugster [pairs of soles]	1	1	1

Currently available resources are:

- 10,000 sq.ft. of NOwater at \$20 per sq.ft.,
- up to 5,000 oz. of Fabrínsula at \$15 per oz.,
- 7,000 pairs of Lugster soles at \$10 per pair.

The manufacturer has already accepted minimum orders of:

- 3,000 pairs of *Walker* boots,
- 2,000 pairs of *Hicker* boots,
- 1,000 pairs of *Backpacker* boots.

If more pairs are produced, buyers are willing to purchase them. The agreed selling prices are \$40, \$65, and \$110 per pair for *Walker*, *Hicker*, and *Backpacker* boots, respectively.

The analyst formulated the following Linear Programming (LP) model to determine the production plan:

$$
\begin{aligned}
\max \quad & z = 12.5x_1 + 25x_2 + 30x_3 & \\
\text{s. t.:} \quad & \frac{1}{2}x_1 + x_2 + \frac{5}{2}x_3 \leq 10\,000 & \text{[NOwater]} \\
& \frac{1}{2}x_1 + \frac{2}{3}x_2 + \frac{4}{3}x_3 \leq 5\,000 & \text{[Fabrínsula]} \\
& x_1 + x_2 + x_3 \leq 7\,000 & \text{[Lugster]} \\
& x_1 \geq 3\,000 & \text{[minimum Walker]} \\
& x_2 \geq 2\,000 & \text{[minimum Hicker]} \\
& x_3 \geq 1\,000 & \text{[minimum Backpacker]} \\
& x_1, x_2, x_3 \geq 0 &
\end{aligned}
$$

The results obtained from solving this model are shown in the Tables 3 and 4.

Provide the answers that the analyst should give to the following questions posed by the company manager.

Q1. How many pairs of boots should we produce? What is the profit?

Tabela 3: Answer Report.

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$E$3	Max	0	143750

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$2	Walker	0	3000	Contin
$C$2	Hicker	0	2750	Contin
$D$2	Backpacker	0	1250	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$5	NOWater	7375	$E$5<=$G$5	Not Binding	2625
$E$6	Fabrinsula	5000	$E$6<=$G$6	Binding	0
$E$7	Lugster	7000	$E$7<=$G$7	Binding	0
$E$8	Min Walker	3000	$E$8>=$G$8	Binding	0
$E$9	Min Hicker	2750	$E$9>=$G$9	Not Binding	750
$E$10	Min Backpacker	1250	$E$10>=$G$10	Not Binding	250

Tabela 4: Sensitivity Report.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$2	Walker	3000	0	12,5	11,25	1E+30
$C$2	Hicker	2750	0	25	5	9
$D$2	Backpacker	1250	0	30	20	5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$5	NOWater	7375	0	10000	1E+30	2625
$E$6	Fabrinsula	5000	7,5	5000	500	166,6666667
$E$7	Lugster	7000	20	7000	250	375
$E$8	Min Walker	3000	-11,25	3000	600	1000
$E$9	Min Hicker	2750	0	2000	750	1E+30
$E$10	Min Backpacker	1250	0	1000	250	1E+30

Q2. You know that NOwater, Fabrínsula, and Lugster are critical resources. How much will we use in this plan, and how much will remain?

Q3. The marketing manager told me that *Hicker* boots are in high demand and that we could increase the price by $10 per pair. Would this increase change the plan?

Q4. We are producing 3,000 pairs of *Walker* boots to fulfill our current order. I wonder whether it would be advantageous to lower the price. Would sales increase?

Q5. The other day I saw an advertisement from a supplier selling Lugster soles at $19 per pair. Do you think it is worth contacting them?