STATISTICS I

Bachelor's degrees in Economics and Finance

2nd Year/2nd Semester

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Pratical Class 2

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https://doity.com.br/estatistica-aplicada-a-nutricao

https://basiccode.com.br/produto/informatica-basica/

Conditional Probability, Total Probability Theorem and Bayes´s Theorem: Exercises

1

Conditional Probability

Sejam A e B acontecimentos associados à mesma experiência aleatória. A probabilidade condicional de A dado que se observou B (ou probabilidade condicional de A se B), é o valor do quociente:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \text{ com } P(B) \neq 0$$

$$P(A \cap B) = P(A|B) \times P(B).$$

Da mesma forma, tem-se

$$P(B \mid A) = \frac{P(B \cap A)}{P(A)}.$$

$$P(A \cap B) = P(B|A) \times P(A).$$

Além disso, $P(\overline{A}|B) = 1 - P(A|B)$.

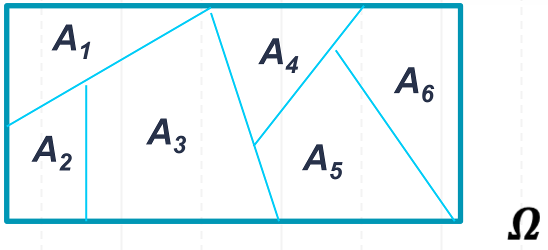
Independent Events

$$P(A \cap B) = P(A) \times P(B)$$

Partition

Diz-se que os acontecimentos $A_1, A_2, \ldots, A_n$ constituem uma partição de Ω se e só se:

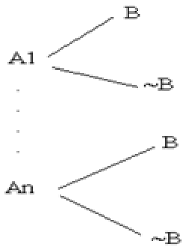
- $A_1, A_2, \ldots, A_n$ são mutuamente incompatíveis, isto é,
  $A_i \cap A_j = \varnothing;\ i, j = 1, 2, \ldots, n;\ i \neq j$;
- $A_1 \cup A_2 \cup \ldots \cup A_n = \Omega$.

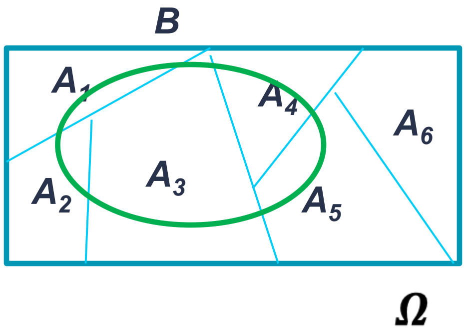


Total Probability Theorem

Se $A_1, A_2, \ldots, A_n$ definem uma partição sobre Ω, então para qualquer B definido em Ω temos:

$$P(B) = \sum_{i=1}^{n} P[B|A_i] \times P(A_i) = P[B|A_1] \times P(A_1) + P[B|A_2] \times P(A_2) + \ldots + P[B|A_n] \times P(A_n)$$





Se a partição é A, $\bar{A}$, então tem-se o caso especial

$$P(B)=P(B|A)P(A)+P(B|\bar{A})P(\bar{A})$$

Bayes' Theorem

Se $A_1, A_2, \ldots, A_n$ definem uma partição sobre Ω, então para qualquer B definido em Ω temos:

$$P[A_i|B] = \frac{P[B|A_i] \times P(A_i)}{\sum_{i=1}^{n} P[B|A_i] \times P(A_i)} = \frac{P(B|A_i)P(A_i)}{P(B|A_1)P(A_1) + \ldots + P(B|A_n)P(A_n)} \quad i=1,2,\ldots,n.$$

Total Probability Theorem

Quando a partição de A é $\overline{A}$, tem-se o caso especial

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\overline{A})P(\overline{A})}.$$

14. The weather forecast says that tomorrow it will rain with probability 0.4. Additionally, according to the weather forecast, the probability that:

- it will rain tomorrow and the day after is 0.2;
- it will rain tomorrow and it will be cloudy in the day after is 0.1.

a) What is the probability that it will rain the day after tomorrow given that it rains tomorrow?

b) What is the probability that it will be cloudy the day after tomorrow given that it rains tomorrow?

Exercise 14 a)

$A = \{$ It rains tomorrow $\}$

$B = \{$ It rains the day after tomorrow $\}$

$C = \{$ It is cloudy in the day after tomorrow

$P(A) = 0.4$ $\qquad$ $P(A \cap B) = 0.2$

$P(A \cap C) = 0.1$

a)

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.4} = \frac{1}{2}$$

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Exercise 14 b)

b)

$$P(C \mid A) = \frac{P(C \cap A)}{P(A)} = \frac{P(A \cap C)}{P(A)} = \frac{0.1}{0.4} = \frac{1}{4}$$

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16. A factory uses three machines to produce the same product. Machines A, B and C produce respectively 40%, 35% and 25% of total production. The percentage of defective parts produced by each machine are respectively 4%, 2% and 1%. If a piece is randomly selected from the total production.

a) What is the probability that it is not defective?

b) Knowing that it is defective what is the probability that it has been produced by machine A?

c) If two pieces are successively removed with replacement from the total production, what is the probability that one of them be defective and the other not?

Exercise 16

$M_A \equiv \{$ The piece is produced by machine A $\}$

$M_B \equiv \{$ " " " " " " B $\}$

$M_C \equiv \{$ " " " " " " C $\}$

$D \equiv \{$ The piece is defective $\}$

$P(M_A) = 0.4 \qquad P(M_B) = 0.35 \qquad P(M_C) = 0.25$

$P(D|M_A) = 0.04 \qquad P(D|M_B) = 0.02 \qquad P(D|M_C) = 0.01$

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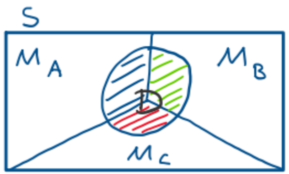
Exercise 16 a)

a)



$D = (M_A \cap D) \cup (M_B \cap D) \cup (M_C \cap D)$

$P(M_A \cap D)$
$P(M_B \cap D)$
$P(M_C \cap D)$

$M_A \cup M_B \cup M_C = S$

$M_i \cap M_j = \emptyset \quad (i, j = A, B, C \wedge i \neq j)$

M_A, M_B and M_C are a partition of S.

We can therefore use the rule of total probability:

$$P(D) = P(M_A \cap D) + P(M_B \cap D) + P(M_C \cap D) =$$

$$= P(M_A)\,P(D|M_A) + P(M_B)\,P(D|M_B) + P(M_C)\,P(D|M_C) =$$

$$= 0.4 \times 0.04 + 0.35 \times 0.02 + 0.25 \times 0.01 =$$

$$= 0.0255$$

$$P(\bar{D}) = 1 - P(D) = 1 - 0.0255 = 0.9745$$

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Exercise 16 b)

b)

$$P(M_A \mid D) = \frac{P(M_A \cap D)}{P(D)} = \frac{P(M_A)\,P(D \mid M_A)}{P(D)} =$$

$$= \frac{0.4 \times 0.04}{0.0255} = 0.6275$$

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Exercise 16 c)

c)

$A \equiv \{$ One of the pieces is defective and the other not $\}$

$\bar{D}\ D$ / $D\ \bar{D}$ $\Big\}$ 2 possibilities

$P(A) = P(\bar{D})P(D) + P(D)P(\bar{D}) =$

$= 2\,P(\bar{D})\,P(D) = 2 \times 0.0255 \times 0.9745 =$

$= 0.0497$

or

$\underbrace{\binom{2}{1}}_{\text{Distinguishable permutations of 2 elements of 2 types (Binomial coefficient)}} \underbrace{P(\bar{D})\,P(D)}_{\text{Probability of one specific ordering}} = 2 \times 0.0255 \times 0.9745 = 0.0497$

Probability of one specific ordering

Distinguishable permutations of 2 elements of 2 types (Binomial coefficient)

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17. In a course of Statistics, students have to complete an exam in two hours. In the set of students that deliver the exam, 20% of them deliver it before the 2 hours, 50% of them deliver it on time and the remaining ones after the 2 hours. From the first ones, 70% have positive grade, from the second ones, 50% have positive grade, and, from the last ones 15% have positive grade.

a) What is the percentage of students that have positive grade?

b) Comment the following sentence: "in the set of students that have positive grade, more than half of them have delivered the exam on the regulatory time".

c) Choose randomly 10 students that have delivered the exam. What is the probability that 4 of them have delivered it on time?

Exercise 17

$A \equiv \}$ Student delivers exam before the 2 hours $\{$
$B \equiv \}$ " " " on time $\{$
$C \equiv \}$ " " after the 2 hours $\{$
$D \equiv \}$ " has positive grade $\{$

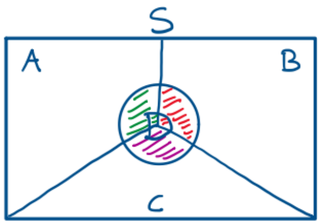
delivery on regulatory time

$P(A) = 0.2$ $\qquad$ $P(D|A) = 0.7$

$P(B) = 0.5$ $\qquad$ $P(D|B) = 0.5$

$P(C) = 1 - 0.2 - 0.5 = 0.3$ $\qquad$ $P(D|C) = 0.15$

A, B and C are a partition of S:



$A \cup B \cup C = S$

$A \cap B = B \cap C = A \cap C = \emptyset$

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Exercise 17 a)

a)

$$P(D) = P(A)\,P(D|A) + P(B)\,P(D|A) + P(C)\,P(D|C) =$$

$$= 0.2 \times 0.7 + 0.5 \times 0.5 + 0.3 \times 0.15 = 0.435$$

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Exercise 17 b)

$A \cup B \equiv \{$ Student delivers the exam on regulatory time $\}$

$$P(A \cup B \mid D) = \frac{P((A \cup B) \cap D)}{P(D)} = \frac{P((A \cap D) \cup (B \cap D))}{P(D)} =$$

Because $(A \cap D) \cap (B \cap D) = \emptyset$

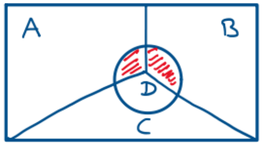
$P(A \cap D)$

$P(B \cap D)$

$$= \frac{P(A \cap D)}{P(D)} + \frac{P(B \cap D)}{P(D)} =$$

$$= \frac{P(A)\,P(D \mid A)}{P(D)} + \frac{P(B)\,P(D \mid B)}{P(D)} =$$

$$= \frac{0.2 \times 0.7}{0.435} + \frac{0.5 \times 0.5}{0.435} = 0.8966$$



$= (A \cup B) \cap D = (A \cap D) \cup (B \cap D)$

Remark: Since $A \cap B = \emptyset$ we could have used:

$$P(A \cup B \mid D) = P(A \mid D) + P(B \mid D) =$$

$$= \frac{P(A)\,P(D \mid A)}{P(D)} + \frac{P(B)\,P(D \mid B)}{P(D)}$$

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Exercise 17 c)

c)

$$\binom{10}{4} 0.5^4 (1-0.5)^{10-4} \simeq 0.205$$

Binomial coefficient (for distinguishable permutations of 10 students where 4 delivered on time and 6 students didn't.

Probability of one specific ordering of 4 students that delivered on time and 6 that didn't.

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18. Let A and B be two events in the sample space S, such that $P(A) \neq 0$ and $P(B) \neq 0$. Prove that if $P(A|B) \geq P(A)$ then $P(B|A) \geq P(B)$.

Exercise 18

$A \subset S \qquad B \subset S$

$P(A) \neq 0 \qquad P(B) \neq 0$

If $P(A|B) \geqslant P(A)$ then:

$$\frac{P(A \cap B)}{P(B)} \geqslant P(A) \Leftrightarrow P(A \cap B) \geqslant P(A)P(B)$$

In that case we have:

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \geqslant \frac{P(A)\,P(B)}{P(A)} = P(B)$$

Conclusion:

If $P(A|B) \geqslant P(A)$ then $P(B|A) \geqslant P(B)$, QED

Alternative solution:

$$P(A|B) \geqslant P(A) \Leftrightarrow$$

$$\Leftrightarrow \frac{P(A \cap B)}{P(B)} \geqslant P(A) \Leftrightarrow$$

$$\Leftrightarrow \frac{P(A \cap B)}{P(A)} \geqslant P(B) \Leftrightarrow P(B|A) \geqslant P(B), \text{ QED}$$

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19. At a production process, the produced items are tested for defects. A defective unit is classified as such with probability 0.9, whereas a correct unit is classified as such with probability 0.85. Furthermore, 10% of the produced units are defective.

a) Define the events and, by using these events, present all the probabilities mentioned in the description of the problem.

b) Compute (i) the probability that a unit is correct and (ii) the probability that a unit is correct but classified as defective.

c) Compute the conditional probability that a unit is defective, given that it has been classified as such.

Exercise 19 a)

a)

$D \equiv$ {The product is defective}

$\bar{D} \equiv$ { " " " not defective } = { The product is correct }

$I \equiv$ { " " " classified as defective }

$P(I \mid D) = 0.9$ $\quad P(D) = 0.1$ $\quad P(\bar{D}) = 1 - 0.1 = 0.9$

$P(\bar{I} \mid \bar{D}) = 0.85$ $\quad P(I \mid \bar{D}) = 1 - 0.85 = 0.15$

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Exercise 19 b)

b)

(i) $P(\bar{D}) = 1 - P(D) = 0.9$

(ii) $P(\bar{D} \cap I) = P(\bar{D})P(I \mid \bar{D}) =$

$= P(\bar{D})(1 - p(\bar{I} \mid \bar{D})) =$

$= 0.9(1 - 0.85) = 0.135$

Solutions say 0.153.
Should be a typo

c)

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Exercise 19 c)

$$P(D|I) = \frac{P(D \cap I)}{P(I)} = \frac{P(I|D)\,P(D)}{P(I)}$$

$$= \frac{0.9 \times 0.1}{0.225} = 0.4$$

<u>Auxiliary calculations</u>:

Because D and $\bar{D}$ are a partition of S

$$P(I) = P(I \cap D) + P(I \cap \bar{D}) =$$

$$= P(D)\,P(I|D) + P(\bar{D})\,P(I|\bar{D}) =$$

$$= 0.1 \times 0.9 + 0.9 \times 0.15 = 0.225$$

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20. Due to an economic downturn, a local factory is forced to lay off workers. The board of the factory has provided the following information:

- There are 1000 workers: 300 are managers and 700 are non-managers;
- The probability that a worker will be laid off is 0.25;
- The probability that a worker will be laid off given that he/she is a non-manager is 0.3.

a) Define the events and, by using these events, present all the probabilities mentioned in the description of the problem.

b) What is the probability that a worker will be laid off given that he/she is a manager.

c) According with the information provided by the board, compute the number of managers and non-managers that will be laid off and those that will not be laid off.

Exercise 20 a)

a)

1000 workers → 300 managers

1000 workers → 700 non-managers

$A \equiv \{$ The worker is laid off $\}$

$B \equiv \{$ 〃 〃 〃 a manager $\}$

$P(A) = 0.25$ $\qquad$ $P(\bar{A}) = 0.75$

$P(B) = \dfrac{300}{1000} = \dfrac{3}{10} = 0.3$ $\qquad$ $P(\bar{B}) = \dfrac{7}{10} = 0.7$

$P(A \mid \bar{B}) = 0.3$

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Exercise 20 b)

$$P(A) = P(A \cap B) + P(A \cap \bar{B}) \Leftrightarrow$$

$$0.25 = P(B)\,P(A \mid B) + P(\bar{B})\,P(A \mid \bar{B}) \Leftrightarrow$$

$$0.25 = 0.3\,P(A \mid B) + 0.7 \times 0.3 \Leftrightarrow$$

$$P(A \mid B) = \frac{0.25 - 0.7 \times 0.3}{0.3} = 0.1(3) = \frac{2}{15}$$

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Exercise 20 c)

e)

	laid off	not laid off	Total
managers	40	260	300
non - managers	210	490	700
Total	250	750	1000

Auxiliary calculations:

$\#$ of laid off $= 1000\ P(A) = 1000 \times 0.25 = 250$

$\#$ of not laid off $= 1000\ P(\bar{A}) = 1000 \times 0.75 = 750$

$\#$ of laid off managers $= 1000\ P(A \cap B) = 1000\ P(B)\ P(A|B) =$

$$= 1000 \times 0.3 \times \frac{2}{15} = 40$$

Remark: Solutions from pdf are wrong

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21. An expert in soil perforation trusts that there is water in a certain region with probability 0.9. Additionally, from his experience, when there is water, in 75% of the time, water is not found in the first perforation.

   a) What is the probability that water is found in the first perforation?

   b) In the event that water is not found in the first perforation, what is the probability that there is no water in that region?

Exercise 21 a)

$A = \{$ There is water $\}$

$B = \{$ water is found in the first perforation $\}$

$P(A) = 0.9$ $\quad$ $P(\bar{B} \mid A) = 0.75$

$P(B \mid A) = 1 - P(\bar{B} \mid A) = 0.25$

a)

A and $\bar{A}$ are a partition of S. We can use the rule of total probability:

$$P(B) = P(B \cap A) + \underbrace{P(B \cap \bar{A})}_{=0} =$$

(if there is no water we can't find water)

$$= P(A)\,P(B \mid A) = 0.9 \times (1 - 0.75)$$

$$= 0.225$$

$=1$ (Given that there

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Exercise 21 b)

$= 0.225$

b)

$$P(\bar{A} \mid \bar{B}) = \frac{P(\bar{A})\,\overbrace{P(\bar{B} \mid \bar{A})}^{=1}}{P(\bar{B})} = \frac{(1-0.9) \times 1}{1-0.225} =$$

$=1$ (Given that there is no water, the probability not finding water is 1)

$\simeq 0.129$

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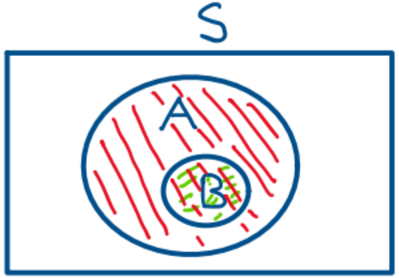
Exercise 21 a)

<u>Other possible solution, noting that B is a subset of A (I prefer this solution to use in the classes):</u>

a)



$$B \subset A$$

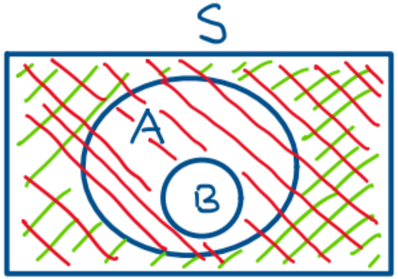
$$B = A \cap B =$$

$$P(B) = P(A \cap B) = P(A)P(B|A)$$

$$= 0.9 \times 0.25 = 0.225$$

Exercise 21 b)

b)



$\bar{A}$

$\bar{B}$

$= \bar{A} \cap \bar{B} = \bar{A}$

$$P(\bar{A} \mid \bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \frac{P(\bar{A})}{P(\bar{B})} = \frac{1 - P(A)}{1 - P(B)} =$$

$$= \frac{1 - 0.9}{1 - 0.225} \simeq 0.129$$

22. Let A, B, C and D be four events in the sample space S, such that A, B and C are a partition of the sample space. Show that if

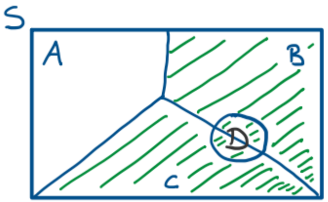
$$D \subset B \cup C \Rightarrow P(D) = P(\overline{A}) \times P(D|\overline{A}).$$

Exercise 22

$A \subset S$
$B \subset S$
$C \subset S$
$D \subset S$

A, B and C are a partition of S:

- $A \cup B \cup C = S$
- $A \cap B = A \cap C = B \cap C = A \cap C = \phi$



$D = D \cap (B \cup C) = D \cap \bar{A}$

$B \cup C = \bar{A}$

If $D \subset (B \cup C)$ then:

$P(D) = P((B \cup C) \cap D) = P(\bar{A} \cap D) =$

$= P(\bar{A})\, P(D \mid \bar{A})$, QED

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23. Let A and B be two independent events in the sample space S such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{3}{4}$.

a) Compute $P(A \cup B)$, $P(A \cap B)$ and $P(B|A \cup B)$;

b) Show that A and $\overline{B}$ are independent events.

Exercise 23 a)

$A \subset S \qquad B \subset S \qquad A \perp B$

$P(A) = \frac{1}{3} \qquad P(B) = \frac{3}{4}$

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) =$

$= P(A) + P(B) - P(A)P(B) =$

$= \frac{1}{3} + \frac{3}{4} - \frac{1}{3} \times \frac{3}{4} =$

$= \frac{4}{12} + \frac{9}{12} - \frac{3}{12} = \frac{10}{12} = \frac{5}{6}$

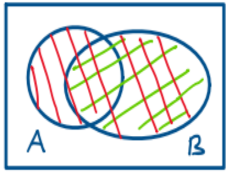
$P(A \cap B) = P(A)P(B) = \frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$

$$P(B \mid A \cup B) = \frac{P(B \cap (A \cup B))}{P(A \cup B)} =$$

$$= \frac{P(B)}{P(A \cup B)} = \frac{3/4}{5/6} = \frac{3}{4} \times \frac{6}{5}$$

$$= \frac{18}{20} = \frac{9}{10}$$

Auxiliary calculations:



$P(B)$

$P(A \cup B)$

$P(B \cap (A \cup B)) = P(B)$

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Exercise 23 b)

b) We need to check if $P(A \cap \bar{B}) = P(A)P(\bar{B})$

$$P(A \cap \bar{B}) = P(A \setminus B) = P(A) - P(A \cap B) =$$
$$= P(A) - P(A)P(B) = P(A)(1 - P(B))$$
$$= P(A)P(\bar{B})$$

<u>Conclusion:</u> A and $\bar{B}$ are independent, QED.

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24. Let A, B and C be three events in the sample space S, such that $P(A \cap B \cap C) = 0.1$, $P(A) = 0.5$, $P(B|A) = 0.4$. Compute the probability $P(C|A \cap B)$.

Exercise 24

$A \subset S$
$B \subset S$
$C \subset S$

$P(A \cap B \cap C) = 0.1$
$P(A) = 0.5$
$P(B|A) = 0.4$

$$P(C \mid A \cap B) = \frac{P((A \cap B) \cap C)}{P(A \cap B)} =$$

$$= \frac{P(A \cap B \cap C)}{P(A)P(B|A)} = \frac{0.1}{0.5 \times 0.4} = \frac{1}{2}$$

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25. Let A and B be two independents events in the sample space S. Supposing that $P(A \cup B) = 0.0298$, $P(A \cap B) = 0.0002$, $P(A) < P(B)$, find $P(A)$ and $P(B)$.

Exercise 25

$A \subset S \quad B \subset S \quad A \perp B$

$P(A \cup B) = 0.0298$

$P(A \cap B) = P(A)\,P(B) = 0.0002$

$P(A) < P(B)$

$$\begin{cases} P(A \cup B) = P(A) + P(B) - P(A \cap B) \\ P(A \cap B) = P(A)\,P(B) \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} 0.0298 = P(A) + P(B) - 0.0002 \\ 0.0002 = P(A)\,P(B) \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} P(A) = 0.03 - P(B) \\ \rule{4em}{0.4pt} \end{cases} \Leftrightarrow \begin{cases} \rule{8em}{0.4pt} \\ (0.03 - P(B))\,P(B) = 0.0002 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \rule{8em}{0.4pt} \\ -P(B)^2 + 0.03\,P(B) - 0.0002 = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \rule{8em}{0.4pt} \\ P(B) = 0.01 \ \vee\ P(B) = 0.02 \end{cases} \Leftrightarrow$$

Because $P(A) < P(B$

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Exercise 25

$$\Leftrightarrow \begin{cases} \rule{10em}{0.4pt} \\ P(B) = 0.01 \vee P(B) = 0.02 \end{cases} \Leftrightarrow$$

Because $P(A) < P(B)$

$$\Leftrightarrow \begin{cases} \cancel{P(A) = 0.03 - 0.01 = 0.02} \\ \cancel{P(B) = 0.01} \end{cases} \vee \boxed{\begin{cases} P(A) = 0.03 - 0.02 = 0.01 \\ P(B) = 0.02 \end{cases}}$$

Auxiliary calculations:

Formula for 2nd degree equation:

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$P(B) = \frac{-0.03 \pm \sqrt{0.03^2 - 4(-1) \times (-0.0002)}}{2(-1)} =$$

$$= \frac{-0.03 \pm \sqrt{0.0001}}{-2} = \frac{-0.03 \pm 0.01}{-2} \Leftrightarrow$$

$$\Leftrightarrow P(B) = \frac{0.02}{2} \vee P(B) = \frac{0.04}{2} \Leftrightarrow$$

$$\Leftrightarrow P(B) = 0.01 \vee P(B) = 0.02$$

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Exercise 25

Alternative resolution:

$$\Leftrightarrow \begin{cases} 0.0002 = P(A)P(B) \\ 0.0298 = P(A) + P(B) - 0.0002 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} P(A) = \dfrac{0.0002}{P(B)} \\ \text{\_\_\_\_\_\_} \end{cases} \Leftrightarrow \begin{cases} \text{\_\_\_\_\_\_} \\ 0.0298 = \dfrac{0.0002}{P(B)} + P(B) - 0.0002 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \text{\_\_\_\_\_\_} \\ 0.0298\,P(B) = 0.0002 + P(B)^2 - 0.0002\,P(B) \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \text{\_\_\_\_\_\_} \\ P(B)^2 - 0.03\,P(B) + 0.0002 = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \text{\_\_\_\_\_\_} \\ P(B) = 0.02 \ \vee \ P(B) = 0.01 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} P(A) = \dfrac{0.0002}{0.02} = 0.01 \\ P(B) = 0.02 \end{cases} \vee \begin{cases} P(A) = \dfrac{0.0002}{0.01} = 0.02 \\ P(B) = 0.01 \end{cases}$$

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Exercise 25

Auxiliary calculations:

$$P(B) = \frac{0.03 \pm \sqrt{(-0.03)^2 - 4(1)(0.0002)}}{2(1)} =$$

$$= \frac{0.03 \pm \sqrt{0.0001}}{2} \Leftrightarrow P(B) = \frac{0.03 \pm 0.01}{2} \Leftrightarrow$$

$$\Leftrightarrow P(B) = 0.02 \vee P(B) = 0.01$$

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Thanks!

Questions?