STATISTICS II

ISEG Lisbon School of Economics & Management
Universidade de Lisboa

Bachelor's degrees in Economics, Finance and Management

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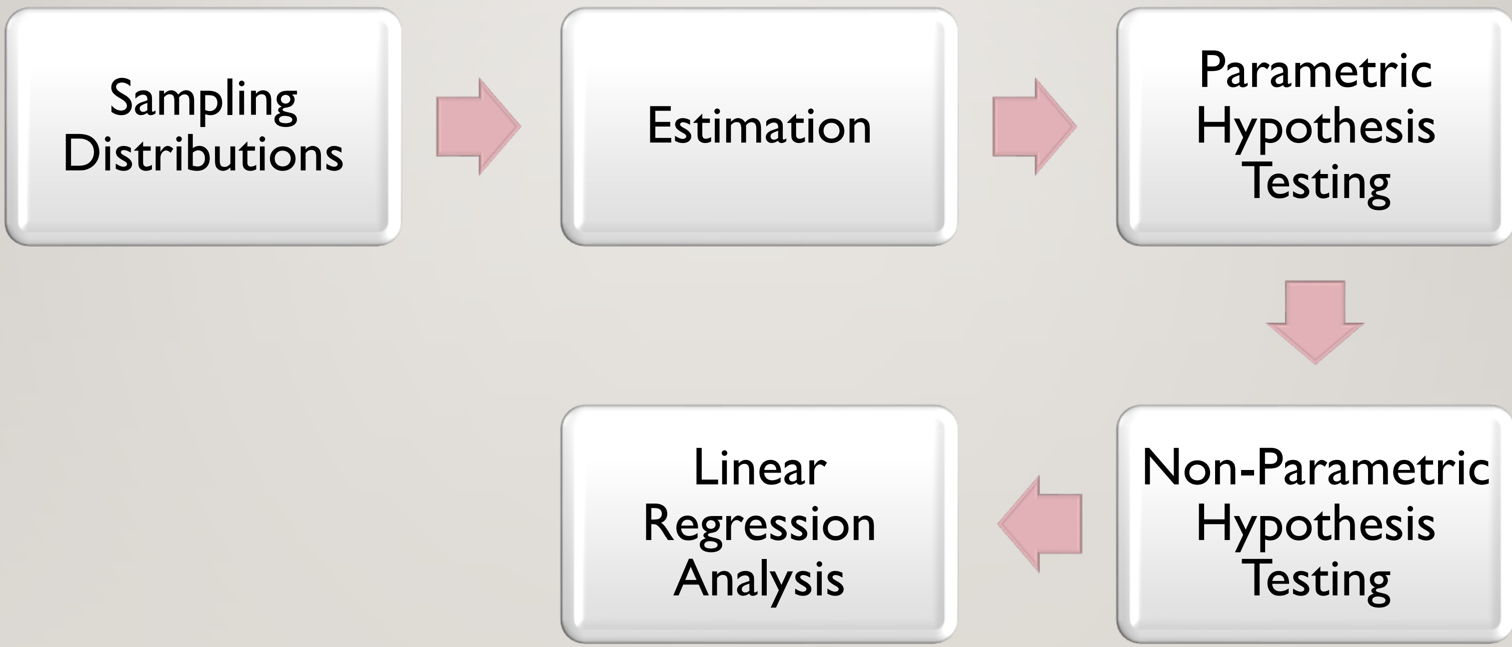
https://doity.com.br/estatistica-aplicada-a-nutricao

https://basiccode.com.br/produto/informatica-basica/

PROGRAM


Sampling Distributions
Estimation
Parametric Hypothesis Testing
Non-Parametric Hypothesis Testing
Linear Regression Analysis

PRATICAL CLASS 4

Exercises 7.41/7.50/7.65/7.68

EXERCISE 7.41

7.41 It is important for airlines to follow the published scheduled departure times of flights. Suppose that one airline that recently sampled the records of 246 flights originating in Orlando found that 10 flights were delayed for severe weather, 4 flights were delayed for maintenance concerns, and all the other flights were on time.

a. Estimate the percentage of on-time departures using a 98% confidence level.
b. Estimate the percentage of flights delayed for severe weather using a 98% confidence level.

Newbold et al (2013)

EXERCISE 7.41 A): SOLUTION

✅ Answer:

Given information

- Total number of flights:

$$n = 246$$

- Flights delayed due to **severe weather**: 10
- Flights delayed due to **maintenance**: 4
- Flights **on time**:

$$246 - (10 + 4) = 232$$

We use the **normal approximation confidence interval for a population proportion**:

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

For a **98% confidence level**:

$$z_{0.01} \approx 2.326$$

EXERCISE 7.41 A): SOLUTION

✅ Answer:

a) 98% confidence interval for the percentage of on-time departures

Sample proportion

$$\hat{p}_{\text{on-time}} = \frac{232}{246} \approx 0.9431$$

Standard error

$$SE = \sqrt{\frac{0.9431(1 - 0.9431)}{246}} \approx 0.0148$$

Margin of error

$$E = 2.326 \times 0.0148 \approx 0.0344$$

Confidence interval

$$0.9431 \pm 0.0344$$

$$(0.9087,\ 0.9775)$$

EXERCISE 7.41 B): SOLUTION

✅ Answer:

b) 98% confidence interval for the percentage of flights delayed due to severe weather

Sample proportion

$$\hat{p}_{\text{weather}} = \frac{10}{246} \approx 0.0407$$

Standard error

$$SE = \sqrt{\frac{0.0407(1 - 0.0407)}{246}} \approx 0.0126$$

Margin of error

$$E = 2.326 \times 0.0126 \approx 0.0293$$

Confidence interval

$$0.0407 \pm 0.0293$$

$$(0.0114,\ 0.0700)$$

EXERCISE 7.50

7.50 A manufacturer bonds a plastic coating to a metal surface. A random sample of nine observations on the thickness of this coating is taken from a week's output, and the thicknesses (in millimeters) of these observations are as follows:

19.8 21.2 18.6 20.4 21.6 19.8 19.9 20.3 20.8

Assuming normality, find a 90% confidence interval for the population variance.

Newbold et al (2013)

EXERCISE 7.50: SOLUTION

Answer:

Given:

Sample data (thickness in mm):

$$19.8, 21.2, 18.6, 20.4, 21.6, 19.8, 19.9, 20.3, 20.8$$

Sample size: $n = 9$

Confidence level: 90% → $\alpha = 0.10$

We want a **CI for** σ^2.

Step 1c: Compute sample variance

$$s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1} = \frac{6.299}{8} \approx 0.7874$$

Step 2: Confidence interval formula

For a **normal population**, the CI for the **variance** uses the **chi-square distribution**:

$$\frac{(n-1)s^2}{\chi^2_{1-\alpha/2}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{\alpha/2}}$$

- $n - 1 = 8$
- Confidence level = 90% → $\alpha = 0.10$
  $\alpha/2 = 0.05$

From chi-square tables:

$$\chi^2_{0.95,8} \approx 15.507 \quad , \quad \chi^2_{0.05,8} \approx 2.180$$

EXERCISE 7.50: SOLUTION

✅ Answer:

Step 3: Compute the CI

Lower limit:

$$\frac{(n-1)s^2}{\chi^2_{1-\alpha/2}} = \frac{8 \times 0.7874}{15.507} \approx \frac{6.299}{15.507} \approx 0.406$$

Upper limit:

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} = \frac{6.299}{2.180} \approx 2.888$$

Step 4: Confidence interval

$$\boxed{0.406 \leq \sigma^2 \leq 2.888}$$

- **Optional:** CI for the standard deviation σ:

$$\sqrt{0.406} \leq \sigma \leq \sqrt{2.888} \implies 0.637 \leq \sigma \leq 1.700$$

✅ **Answer:**

90% confidence interval for the population variance:

$$0.406 \leq \sigma^2 \leq 2.888$$

Optional: For the standard deviation:

$$0.64 \leq \sigma \leq 1.70$$

EXERCISE 7.65

7.65 How large of a sample is needed to estimate the mean of a normally distributed population for each of the following?

a. $ME = 5; \sigma = 40; \alpha = 0.01$
b. $ME = 10; \sigma = 40; \alpha = 0.01$
c. Compare and comment on your answers to parts a and b.

Newbold et al (2013)

EXERCISE 7.65 A): SOLUTION

✅ Answer:

When estimating the mean of a normally distributed population **with known population standard deviation** σ, the required sample size is

$$n = \left(\frac{z_{1-\alpha/2}\,\sigma}{\mathrm{ME}}\right)^2$$

For $\alpha = 0.01$:

$$z_{1-0.01/2} = z_{0.995} \approx 2.576$$

a. $\mathrm{ME} = 5,\ \ \sigma = 40,\ \ \alpha = 0.01$

$$n = \left(\frac{2.576 \times 40}{5}\right)^2 = (20.608)^2 \approx 424.7$$

$$\boxed{n = 425}$$

EXERCISE 7.65 B): SOLUTION

✅ Answer:

When estimating the mean of a normally distributed population **with known population standard deviation** σ, the required sample size is

$$n = \left(\frac{z_{1-\alpha/2}\,\sigma}{\mathrm{ME}}\right)^2$$

For $\alpha = 0.01$:

$$z_{1-0.01/2} = z_{0.995} \approx 2.576$$

b. $\mathrm{ME} = 10,\ \sigma = 40,\ \alpha = 0.01$

$$n = \left(\frac{2.576 \times 40}{10}\right)^2 = (10.304)^2 \approx 106.2$$

$$\boxed{n = 107}$$

EXERCISE 7.65 C): SOLUTION

✅ Answer:

c. Comparison and comment

- Doubling the margin of error from 5 to 10 reduces the required sample size by a factor of **four**.
- This occurs because the sample size is **inversely proportional to the square of the margin of error**:

$$n \propto \frac{1}{\mathrm{ME}^2}$$

- Therefore, achieving higher precision requires substantially larger samples.

EXERCISE 7.68

7.68 A research group wants to estimate the proportion of consumers who plan to buy a scanner for their PC during the next 3 months.

a. How many people should be sampled so that the sampling error is at most 0.04 with a 90% confidence interval?
b. What is the sample size required if the confidence is increased to 95%, keeping the sampling error the same?
c. What is the required sample size if the research group extends the sampling error to 0.05 and wants a 98% confidence level?

Newbold et al (2013)

EXERCISE 7.68 A): SOLUTION

✅ Answer:

General sample size formula (for proportions)

$$n = \left(\frac{z_{1-\alpha/2}}{\text{ME}}\right)^2 p(1-p)$$

with $p(1-p) = 0.25$.

a. Sampling error $= 0.04$, confidence level $= 90\%$

- $\alpha = 0.10$
- $z_{1-\alpha/2} = z_{0.95} \approx 1.645$

$$n = \left(\frac{1.645}{0.04}\right)^2 (0.25)$$

$$n = (41.125)^2 \times 0.25 \approx 423.1$$

$$\boxed{n = 424}$$

EXERCISE 7.68 B): SOLUTION

Answer:

General sample size formula (for proportions)

$$n = \left(\frac{z_{1-\alpha/2}}{\text{ME}}\right)^2 p(1-p)$$

with $p(1-p) = 0.25$.

b. Sampling error $= 0.04$, confidence level $= 95\%$

- $\alpha = 0.05$
- $z_{1-\alpha/2} = z_{0.975} \approx 1.96$

$$n = \left(\frac{1.96}{0.04}\right)^2 (0.25)$$

$$n = (49)^2 \times 0.25 = 600.25$$

$$\boxed{n = 601}$$

EXERCISE 7.68 C): SOLUTION

✅ Answer:

General sample size formula (for proportions)

$$n = \left(\frac{z_{1-\alpha/2}}{\text{ME}}\right)^2 p(1-p)$$

with $p(1-p) = 0.25$.

c. Sampling error $= 0.05$, confidence level $= 98\%$

- $\alpha = 0.02$
- $z_{1-\alpha/2} = z_{0.99} \approx 2.326$

$$n = \left(\frac{2.326}{0.05}\right)^2 (0.25)$$

$$n = (46.52)^2 \times 0.25 \approx 541.1$$

$$\boxed{n = 542}$$

Key observations

- Increasing the **confidence level** increases the required sample size.
- Increasing the **sampling error** decreases the required sample size.
- The relationship with the margin of error is **quadratic**:

$$n \propto \frac{1}{\text{ME}^2}$$

THANKS!

Questions?