STATISTICS II

Bachelor's degrees in Economics, Finance and Management

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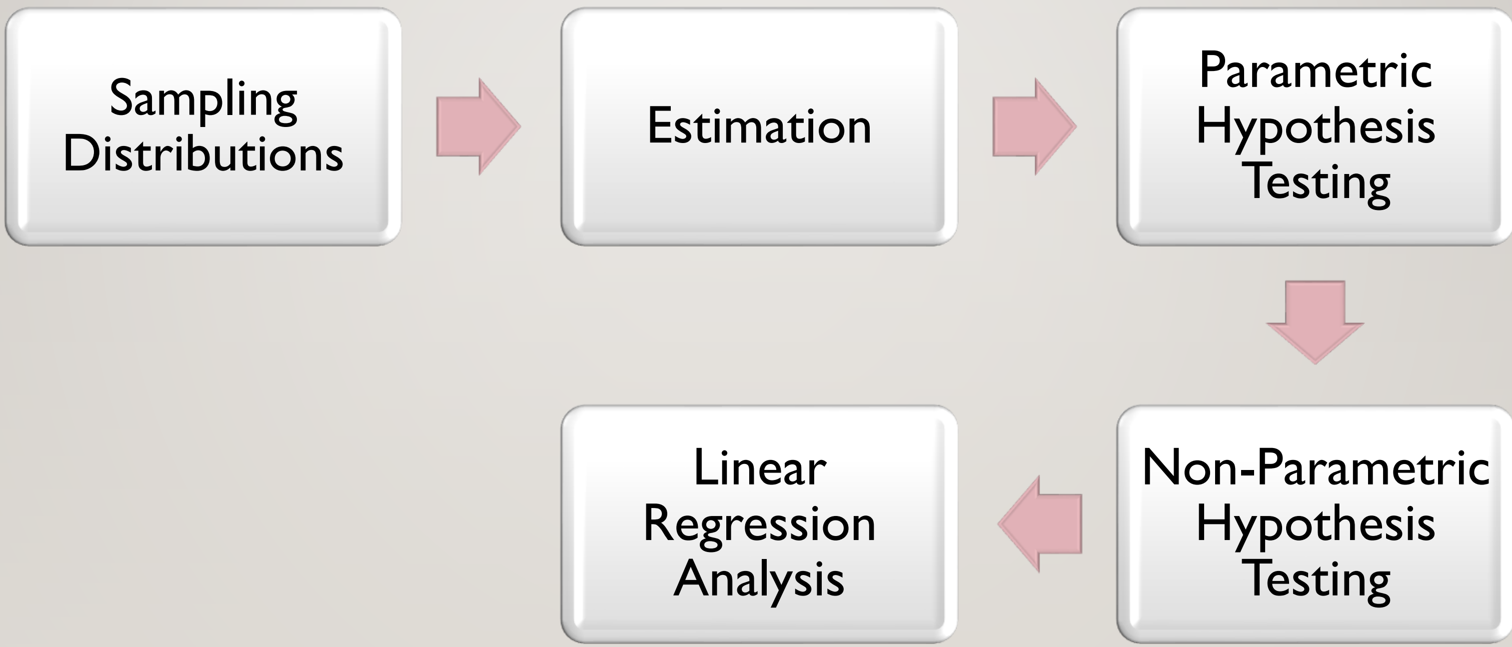
https://doity.com.br/estatistica-aplicada-a-nutricao

https://basiccode.com.br/produto/informatica-basica/

PROGRAM


Sampling Distributions
Estimation
Parametric Hypothesis Testing
Non-Parametric Hypothesis Testing
Linear Regression Analysis

PRATICAL CLASS 8

Exercises 9.4, 9.5, 9.7 A) and B), 9.9 A) and B), 9.13

EXERCISE 9.4

9.4 In the UK, some motorist groups want the current speed limit on motorways increased; they argue this would not be dangerous and would enable motorists to reach their destinations more quickly. However, some road-safety groups say speed can be a factor in accidents and believe it would be dangerous to increase the existing speed limit.

a. State the null and alternative hypotheses from the perspective of the motorist groups.
b. State the null and alternative hypotheses from the perspective of road-safety groups.

Newbold et al (2013)

EXERCISE 9.4 A): SOLUTION

Answer:

a) From the perspective of the motorist groups

Motorist groups believe that **increasing the speed limit would not increase danger** (i.e., it would not increase accidents).

Let μ = mean number of accidents (or accident rate) after increasing the speed limit.
Let μ_0 = current mean number of accidents.

Motorists want to show that accidents **do not increase**.

$$H_0 : \mu \geq \mu_0$$

$$H_1 : \mu < \mu_0$$

Interpretation:

- H_0: Increasing the speed limit is dangerous (accidents stay the same or increase).
- H_1: Increasing the speed limit is not dangerous (accidents decrease).

EXERCISE 9.4 B): SOLUTION

✅ Answer:

b) From the perspective of road-safety groups

Road-safety groups believe that **increasing the speed limit would increase accidents**.

$$H_0 : \mu \leq \mu_0$$

$$H_1 : \mu > \mu_0$$

Interpretation:

- H_0: Increasing the speed limit does not increase accidents.
- H_1: Increasing the speed limit increases accidents (more dangerous).

EXERCISE 9.5

9.5 The branch manager of an international bank in Kuala Lumpur, Malaysia, has received a memorandum from senior executives at the head office of the bank instructing the manager to ensure that the average queuing time for customers waiting to see a cashier is no more than 5 minutes. Since receiving this directive, the manager has been informally checking queuing times and is very confident that the average time customers spend waiting to see a cashier is currently 5 minutes or less. You have now been brought in to undertake an audit of queuing times to check that they are in accordance with the senior executives' directive. State the null and alternative hypotheses you will be using in this instance.

Newbold et al (2013)

EXERCISE 9.5: SOLUTION

✅ Answer:

Let μ be the **mean queuing time (in minutes)** for customers waiting to see a cashier.

The directive from senior management states that the **average waiting time must be no more than 5 minutes**. Therefore, the claim that needs to be verified is that the mean waiting time **does not exceed 5 minutes**.

Hypotheses

$$H_0 : \mu \leq 5$$

$$H_1 : \mu > 5$$

Interpretation

- H_0: The average waiting time is **5 minutes or less** (the bank is complying with the directive).
- H_1: The average waiting time is **greater than 5 minutes** (the directive is not being met).

This corresponds to a **right-tailed test**, because we are testing whether the mean waiting time **exceeds** 5 minutes.

EXERCISE 9.7 A) AND B)

9.7 A random sample is obtained from a population with variance $\sigma^2 = 625$, and the sample mean is computed. Test the null hypothesis $H_0: \mu = 100$ versus the alternative hypothesis $H_1: \mu > 100$ with $\alpha = 0.05$. Compute the critical value $\bar{x}_c$ and state your decision rule for the following options.

a. Sample size $n = 25$
b. Sample size $n = 16$

Newbold et al (2013)

EXERCISE 9.7: SOLUTION

Answer:

Given

- $\sigma^2 = 625 \Rightarrow \sigma = 25$
- $H_0 : \mu = 100$
- $H_1 : \mu > 100$
- $\alpha = 0.05$

This is a **right-tailed test**.

The test statistic is

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

The **critical value for the sample mean** $\bar{x}_c$ is obtained from

$$\bar{x}_c = \mu_0 + z_{1-\alpha} \frac{\sigma}{\sqrt{n}}$$

For $\alpha = 0.05$:

$$z_{0.95} = 1.645$$

Decision rule:

$$\text{Reject } H_0 \text{ if } \bar{X} > \bar{x}_c$$

EXERCISE 9.7 A): SOLUTION

✅ Answer:

a) $n = 25$

$$\frac{\sigma}{\sqrt{n}} = \frac{25}{\sqrt{25}} = \frac{25}{5} = 5$$

$$\bar{x}_c = 100 + 1.645(5)$$

$$\bar{x}_c = 100 + 8.225 = 108.225$$

Decision rule

Reject H_0 if

$$\bar{X} > 108.225$$

EXERCISE 9.7 B): SOLUTION

✅ Answer:

b) $n = 16$

$$\frac{\sigma}{\sqrt{n}} = \frac{25}{\sqrt{16}} = \frac{25}{4} = 6.25$$

$$\bar{x}_c = 100 + 1.645(6.25)$$

$$\bar{x}_c = 100 + 10.281 = 110.28$$

Decision rule

Reject H_0 if

$$\bar{X} > 110.28$$

✅ **Key idea:**

With a **larger sample size**, the standard error is smaller, so the **critical sample mean needed to reject** H_0 is closer to 100.

EXERCISE 9.9 A) AND B)

9.9 A random sample is obtained from a population with a variance of $\sigma^2 = 400$, and the sample mean is computed to be $\bar{x}_c = 70$. Consider the null hypothesis $H_0 : \mu = 80$ versus the alternative hypothesis $H_1 : \mu < 80$. Compute the p-value for the following options.

a. Sample size $n = 25$
b. Sample size $n = 16$

Newbold et al (2013)

EXERCISE 9.9 A): SOLUTION

✅ Answer:

Given

- $\sigma^2 = 400 \Rightarrow \sigma = 20$
- $\bar{x} = 70$
- $\mu_0 = 80$

Hypotheses:

$$H_0 : \mu = 80$$

$$H_1 : \mu < 80$$

This is a **left-tailed test**.

Test statistic:

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

a) $n = 25$

Standard error:

$$\sigma/\sqrt{n} = 20/\sqrt{25} = 20/5 = 4$$

Test statistic:

$$Z_0 = \frac{70 - 80}{4} = -2.5$$

p-value:

$$p\text{-value} = P(Z < -2.5)$$

$$p\text{-value} = 0.0062$$

EXERCISE 9.9 B): SOLUTION

✅ Answer:

Given

- $\sigma^2 = 400 \Rightarrow \sigma = 20$
- $\bar{x} = 70$
- $\mu_0 = 80$

Hypotheses:

$$H_0 : \mu = 80$$

$$H_1 : \mu < 80$$

This is a **left-tailed test**.

Test statistic:

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

b) $n = 16$

Standard error:

$$\sigma/\sqrt{n} = 20/\sqrt{16} = 20/4 = 5$$

Test statistic:

$$Z_0 = \frac{70 - 80}{5} = -2.0$$

p-value:

$$p\text{-value} = P(Z < -2.0)$$

$$p\text{-value} = 0.0228$$

✅ **Results**

- **a)** p-value $= 0.0062$
- **b)** p-value $= 0.0228$

👉 Note: With a **larger sample size**, the standard error decreases, the test statistic becomes more extreme, and the **p-value becomes smaller**.

EXERCISE 9.13

9.13 A pharmaceutical manufacturer is concerned that the impurity concentration in pills should not exceed 3%. It is known that from a particular production run impurity concentrations follow a normal distribution with a standard deviation of 0.4%. A random sample of 64 pills from a production run was checked, and the sample mean impurity concentration was found to be 3.07%.

a. Test at the 5% level the null hypothesis that the population mean impurity concentration is 3% against the alternative that it is more than 3%.
b. Find the p-value for this test.
c. Suppose that the alternative hypothesis had been two-sided, rather than one-sided, with the null hypothesis $H_0 : \mu = 3$. State, without doing the calculations, whether the p-value of the test would be higher than, lower than, or the same as that found in part (b). Sketch a graph to illustrate your reasoning.
d. In the context of this problem, explain why a one-sided alternative hypothesis is more appropriate than a two-sided alternative.

Newbold et al (2013)

EXERCISE 9.13 A): SOLUTION

Answer:

Given

- Population standard deviation: $\sigma = 0.4\%$
- Sample size: $n = 64$
- Sample mean: $\bar{x} = 3.07\%$
- Hypothesized mean: $\mu_0 = 3\%$
- Significance level: $\alpha = 0.05$

Since the population **standard deviation is known** and the distribution is normal, we use a **Z-test**.

a) Hypothesis test

Hypotheses

$$H_0 : \mu \leq 3$$

$$H_1 : \mu > 3$$

This is a **right-tailed test**.

Test statistic

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

$$Z = \frac{3.07 - 3}{0.4/\sqrt{64}}$$

$$Z = \frac{0.07}{0.4/8}$$

$$Z = \frac{0.07}{0.05} = 1.40$$

Critical value

For a **right-tailed test at $\alpha = 0.05$**

$$z_{0.95} = 1.645$$

Decision

$$Z_0 = 1.40 < 1.645$$

$$Z_0 \notin RR$$

Therefore,

Do not reject H_0 at $\alpha = 0.05$.

Conclusion

There is **not enough evidence** at the 5% level to conclude that the mean impurity concentration is greater than 3%.

EXERCISE 9.13 B): SOLUTION

✅ Answer:

b) p-value

$$p\text{-value} = P(Z > 1.40)$$

From the standard normal table:

$$p\text{-value} = 0.0808$$

Since

$$0.0808 > 0.05$$

we **do not reject** H_0.

EXERCISE 9.13 C): SOLUTION

✅ Answer:

c) Two-sided alternative

If the test were

$$H_0 : \mu = 3$$
$$H_1 : \mu \neq 3$$

the **p-value would be larger**.

Reason:

For a two-sided test

$$p\text{-value} = 2P(Z > 1.40)$$

which is approximately

$$2(0.0808) = 0.1616$$

Thus the p-value is **higher**.

Graph explanation

- The **one-sided test** considers only the **right tail**.
- The **two-sided test** considers **both tails**, doubling the probability area.

EXERCISE 9.13 D): SOLUTION

✅ Answer:

d) Why a one-sided test is more appropriate

The pharmaceutical manufacturer is specifically concerned that the impurity concentration **does not exceed** 3%.

Values **below 3% are not problematic**, while values **above 3% violate the quality requirement**.

Therefore, the relevant question is whether the mean impurity concentration **is greater than 3%**, which makes a **one-sided alternative hypothesis** appropriate.

THANKS!

Questions?