STATISTICS II

Bachelor's degrees in Economics, Finance and Management

2nd year/2nd Semester
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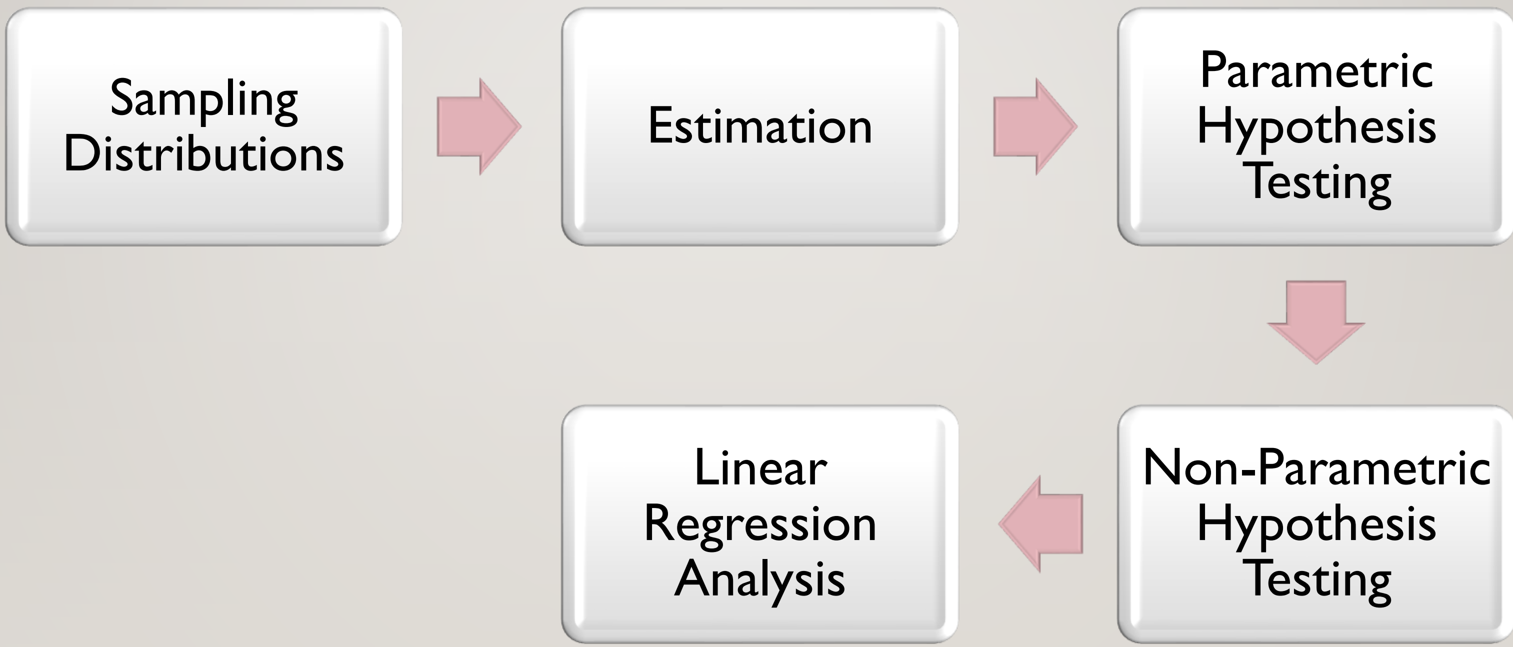
https://doity.com.br/estatistica-aplicada-a-nutricao

https://basiccode.com.br/produto/informatica-basica/

PROGRAM


Sampling Distributions
Estimation
Parametric Hypothesis Testing
Non-Parametric Hypothesis Testing
Linear Regression Analysis

PRATICAL CLASS 9

Exercises 9.24, 9.27, 9.32, 9.34, 9.48 e 9.52

EXERCISE 9.24

9.24 A process that produces bottles of shampoo, when operating correctly, produces bottles whose contents weigh, on average, 20 ounces. A random sample of nine bottles from a single production run yielded the following content weights (in ounces):

21.4 19.7 19.7 20.6 20.8 20.1 19.7 20.3 20.9

Assuming that the population distribution is normal, test at the 5% level against a two-sided alternative the null hypothesis that the process is operating correctly.

Newbold et al (2013)

EXERCISE 9.24: SOLUTION

Answer:

Two-Sided One-Sample *t*-Test (α = 0.05)

We are given a random sample of $n = 9$ shampoo bottles. When operating correctly, the process has mean content $\mu_0 = 20$ ounces. The population distribution is assumed to be normal and the population variance is unknown, so a one-sample *t*-test is appropriate.

Data (ounces):

21.4, 19.7, 19.7, 20.6, 20.8, 20.1, 19.7, 20.3, 20.9

1. Hypotheses

$$H_0 : \ \mu = 20 \quad \text{(process operating correctly)}$$

$$H_1 : \ \mu \neq 20 \quad \text{(two-sided alternative)}$$

EXERCISE 9.24: SOLUTION

Answer:

2. Sample Statistics

- Sample mean:

$$\bar{x} = \frac{183.2}{9} = 20.356 \text{ ounces}$$

- Sample standard deviation:

$$s \approx 0.613 \text{ ounces}$$

3. Test Statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{20.356 - 20}{0.613/\sqrt{9}} \approx 1.74$$

with $df = n - 1 = 8$.

4. Decision Rule

For a two-sided test at the 5% level with 8 degrees of freedom,

$$t_{0.025,8} \approx 2.306.$$

EXERCISE 9.24: SOLUTION

✅ Answer:

5. Decision and Conclusion

Since

$$|t| = 1.74 < 2.306,$$

we **fail to reject** H_0.

(Equivalently, the *p*-value is approximately 0.12, which is greater than 0.05.)

Conclusion

At the 5% significance level, there is **no statistically significant evidence** that the mean content differs from 20 ounces. The data are consistent with the process operating correctly.

EXERCISE 9.27

9.27 In contract negotiations a company claims that a new incentive scheme has resulted in average weekly earnings of at least $400 for all customer service workers. A union representative takes a random sample of 15 workers and finds that their weekly earnings have an average of $381.35 and a standard deviation of $48.60. Assume a normal distribution.

a. Test the company's claim.

b. If the same sample results had been obtained from a random sample of 50 employees, could the company's claim be rejected at a lower significance level than that used in part a?

Newbold et al (2013)

EXERCISE 9.27 A): SOLUTION

✅ Answer:

a) Test the company's claim

Step 1: Hypotheses

Because the claim is "at least $400", this is a **left-tailed test**.

$$H_0 : \ \mu \geq 400$$

$$H_1 : \ \mu < 400$$

Step 2: Given information

- Sample size: $n = 15$
- Sample mean: $\bar{x} = 381.35$
- Sample standard deviation: $s = 48.60$
- Significance level: $\alpha = 0.05$
- Population is assumed normal → use a t-test

EXERCISE 9.27 A): SOLUTION

Answer:

Step 3: Test statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{381.35 - 400}{48.60/\sqrt{15}} \approx \frac{-18.65}{12.55} \approx -1.49$$

Degrees of freedom:

$$df = 15 - 1 = 14$$

Step 4: Critical value

For a **left-tailed test** with $\alpha = 0.05$ and $df = 14$:

$$t_{0.05,14} \approx -1.761$$

Step 5: Decision

$$t = -1.49 > -1.761$$

We **fail to reject** H_0.

Conclusion (part a)

At the 5% significance level, there is **insufficient evidence** to reject the company's claim that average weekly earnings are at least $400.

EXERCISE 9.27 B): SOLUTION

Answer:

b) What if the sample size were $n = 50$?

Assume:

- $\bar{x} = 381.35$
- $s = 48.60$
- $n = 50$

New test statistic

$$t = \frac{381.35 - 400}{48.60/\sqrt{50}} \approx \frac{-18.65}{6.87} \approx -2.71$$

Degrees of freedom:

$$df = 49$$

EXERCISE 9.27 B): SOLUTION

✅ Answer:

Interpretation

A test statistic of about **−2.71** corresponds to a **much smaller p-value** (well below 0.01).

Conclusion (part b)

Yes. With a sample size of 50, the company's claim **could be rejected at a lower significance level** than in part (a), because increasing the sample size reduces the standard error and provides stronger evidence against the null hypothesis.

Final Summary

- **(a)** With $n = 15$, the claim cannot be rejected at the 5% level.
- **(b)** With $n = 50$, the same sample results would allow rejection of the claim at a **lower significance level**, indicating stronger statistical evidence.

EXERCISE 9.32

9.32 In a random sample of 160 business school students, 72 sample members indicated some measure of agreement with this statement: *Scores on a standardized entrance exam are less important for a student's chance to succeed academically than is the student's high school GPA.* Test the null hypothesis that one-half of all business school graduates would agree with this statement against a two-sided alternative. Find and interpret the *p*-value of the test.

Newbold et al (2013)

EXERCISE 9.32: SOLUTION

Answer:

One-Sample Proportion z-Test (Two-Sided)

We test whether the population proportion of business school students who agree with the statement differs from one-half.

1. Hypotheses

$$H_0:\ p = 0.5$$

$$H_1:\ p \neq 0.5$$

2. Sample Information

- Sample size: $n = 160$
- Number who agree: $x = 72$
- Sample proportion:

$$\hat{p} = \frac{72}{160} = 0.45$$

EXERCISE 9.32: SOLUTION

Answer:

3. Test Statistic

Under H_0, the standard error is

$$SE = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.5(0.5)}{160}} = \sqrt{0.0015625} = 0.0395$$

The z-statistic is

$$z = \frac{\hat{p} - p_0}{SE} = \frac{0.45 - 0.50}{0.0395} \approx -1.27$$

4. p-Value

For a **two-sided test**, the p-value is

$$p\text{-value} = 2P(Z \leq -|z|) = 2P(Z \leq -1.27) \approx 2(0.102) \approx 0.204$$

EXERCISE 9.32: SOLUTION

✅ Answer:

5. Decision and Interpretation

Since the *p*-value (≈ 0.20) is **greater than any common significance level** such as 0.05 or 0.10, we **fail to reject** H_0.

Conclusion

There is **no statistically significant evidence** that the proportion of business school students who agree with the statement is different from one-half.

The *p*-value of about **0.20** means that, if in fact one-half of all such students agree with the statement, there is a 20% chance of observing a sample proportion at least as far from 0.5 as 0.45 (in either direction) purely due to random sampling variability.

EXERCISE 9.34

9.34 A random sample of 50 university admissions officers was asked about expectations in application interviews. Of these sample members, 28 agreed that the interviewer usually expects the interviewee to have volunteer experience doing community projects. Test the null hypothesis that one-half of all interviewers have this expectation against the alternative that the population proportion is larger than one-half. Use $\alpha = 0.05$.

Newbold et al (2013)

EXERCISE 9.34: SOLUTION

✅ Answer:

One-Sample Proportion *z*-Test (Right-Tailed, α = 0.05)

Hypotheses

$$H_0: \ p \leq 0.5$$

$$H_1: \ p > 0.5$$

2. Sample Information

- Sample size: $n = 50$
- Number who agree: $x = 28$
- Sample proportion:

$$\hat{p} = \frac{28}{50} = 0.56$$

EXERCISE 9.34: SOLUTION

Answer:

3. Test Statistic

Under H_0, the standard error is

$$SE = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.5(0.5)}{50}} = \sqrt{0.005} \approx 0.0707$$

The z-statistic is

$$z = \frac{\hat{p} - p_0}{SE} = \frac{0.56 - 0.50}{0.0707} \approx 0.85$$

4. Decision Rule

For a right-tailed test at $\alpha = 0.05$,

$$z_{0.95} = 1.645$$

EXERCISE 9.34: SOLUTION

Answer:

5. Decision and Conclusion

Since

$$z = 0.85 < 1.645,$$

we **fail to reject** H_0.

(Equivalently, the *p*-value is approximately 0.20, which is greater than 0.05.)

Conclusion

At the 5% significance level, there is **insufficient evidence** to conclude that more than half of all university admissions interviewers expect applicants to have volunteer experience in community projects.

EXERCISE 9.48

9.48 At the insistence of a government inspector, a new safety device is installed in an assembly-line operation. After the installation of this device, a random sample of 8 days' output gave the following results for numbers of finished components produced:

618 660 638 625 571 598 639 582

Management is concerned about the variability of daily output and views any variance above 500 as undesirable. Test, at the 10% significance level, the null hypothesis that the population variance for daily output does not exceed 500.

Newbold et al (2013)

EXERCISE 9.48: SOLUTION

✅ Answer:

Chi-Square Test for a Population Variance (Right-Tailed, α = 0.10)

Management considers any **variance above 500** to be undesirable. We test whether the population variance exceeds this value.

1. Hypotheses

$$H_0 :\ \sigma^2 \leq 500$$

$$H_1 :\ \sigma^2 > 500$$

This is a **right-tailed chi-square test**.

EXERCISE 9.48: SOLUTION

✅ Answer:

2. Sample Information

Daily output (8 days):

618, 660, 638, 625, 571, 598, 639, 582

- Sample size: $n = 8$
- Sample mean:

$$\bar{x} = \frac{4931}{8} = 616.375$$

- Sample variance:

$$s^2 \approx 935.4$$

(Computed using $s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1}$)

EXERCISE 9.48: SOLUTION

✅ Answer:

3. Test Statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{7(935.4)}{500} \approx 13.10$$

Degrees of freedom:

$$df = n - 1 = 7$$

4. Critical Value

For a right-tailed test with $\alpha = 0.10$ and $df = 7$:

$$\chi^2_{0.90,7} \approx 12.02$$

5. Decision

$$\chi^2 = 13.10 > 12.02$$

We **reject** H_0.

EXERCISE 9.48: SOLUTION

✅ Answer:

6. *p*-Value (Interpretation)

The test statistic lies between the 90th and 95th percentiles of the χ^2_7 distribution, so

$$0.05 < \text{p-value} < 0.10 \quad \text{(approximately 0.07)}$$

Conclusion

At the 10% significance level, there is **sufficient evidence** to conclude that the population variance of daily output **exceeds 500**.

Thus, management's concern about excessive variability after installing the new safety device is **statistically justified**.

EXERCISE 9.52

9.52 An instructor has decided to introduce a greater component of independent study into an intermediate microeconomics course as a way of motivating students to work independently and think more carefully about the course material. A colleague cautions that a possible consequence may be increased variability in student performance. However, the instructor responds that she would expect less variability. From her records she found that in the past, student scores on the final exam for this course followed a normal distribution with standard deviation 18.2 points. For a class of 25 students using the new approach, the standard deviation of scores on the final exam was 15.3 points. Assuming that these 25 students can be viewed as a random sample of all those who might be subjected to the new approach, test the null hypothesis that the population standard deviation is at least 18.2 points against the alternative that it is lower.

Newbold et al (2013)

EXERCISE 9.52: SOLUTION

Answer:

Chi-Square Test for a Population Standard Deviation (Left-Tailed)

The question is whether the **new teaching approach reduces variability** in student performance.

1. Hypotheses

Because the instructor expects **less variability**, this is a **left-tailed test**.

$$H_0: \ \sigma \geq 18.2 \quad (\text{or equivalently } \sigma^2 \geq 18.2^2)$$

$$H_1: \ \sigma < 18.2 \quad (\text{or } \sigma^2 < 18.2^2)$$

2. Sample Information

- Sample size: $n = 25$
- Sample standard deviation: $s = 15.3$
- Hypothesized standard deviation: $\sigma_0 = 18.2$

EXERCISE 9.52: SOLUTION

✅ Answer:

3. Test Statistic

For a normal population, the test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

$$\chi^2 = \frac{24 \times (15.3)^2}{(18.2)^2} = \frac{24 \times 234.09}{331.24} \approx 16.96$$

Degrees of freedom:

$$df = n - 1 = 24$$

4. Critical Value

For a **left-tailed test** at the 5% significance level with $df = 24$:

$$\chi^2_{0.05,\,24} \approx 13.85$$

5. Decision

$$\chi^2 = 16.96 > 13.85$$

Since the test statistic is **not** in the rejection region, we **fail to reject** H_0.

EXERCISE 9.52: SOLUTION

✅ Answer:

6. *p*-Value

The test statistic lies between the 10th and 25th percentiles of the χ^2_{24} distribution, so

$$\text{p-value} \approx 0.15$$

Conclusion

There is **insufficient statistical evidence** to conclude that the population standard deviation of final exam scores under the new teaching approach is **less than 18.2 points**.

Although the sample standard deviation is smaller, the reduction in variability is **not statistically significant** at conventional significance levels.

THANKS!

Questions?