Lisbon School of Economics & Management
Universidade de Lisboa

Microeconomics
Fall 2024-2025
Midterm
October 2024

Duration: 1.5 hour (90 minutes)

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General Guidelines

- You may use a calculator;
- You may **not** use a programmable calculator;
- You may **not** use notes or books;
- You may have some food and beverages on your desk;
- All other belongings, including phones, must be on the floor;
- You can only leave the room after 30 minutes into the exam and up unto 15 minutes before the exam ends;
- Write all your answers on the blank answer sheets brought by you;
- Write your name and student number on every answer sheet;
- Number all your answer sheets and hand them in in chronological order;
- If a question does not ask for an explanation, there is no need to give one;
- This exam is to be handed in together with your answer sheets;
- Any form of fraud will, at least, imply an invalid grade for this course.

1. Production (5 points)

Let $y = \theta x_1^{\alpha} x_2^{\beta}$ be a production function, where y is the output and x_1 and x_2 are the two inputs.

1.1 Find the technical rate of substitution (TRS).

1.2. What happens to the TRS when $\alpha \to 0$? Provide an economic explanation for this.

1.3. Sketch the isoquant for producing the output level $y = \bar{y}$:

$$\{\, (x_1, x_2) \;\; in\ R_+^2 \;\mid\; \theta x_1^{\alpha} x_2^{\beta} = \bar{y} \,\}$$

1.4. Is the input requirement set for this production function convex? Use your sketch above to provide graphical proof for your answer.

2. Profit (6 points)

Consider a firm that produces output y while using inputs x_1 and x_2. The price for output y is denoted by p and the price for inputs x_1 and x_2 is denoted by w_1 and w_2. Imagine that you observe the following data for the firm across two months:

Month	p	w_1	w_2	y	x_1	x_2
1	4	4	2	10	2	3
2	3	2	3	10	3	2

2.1. Can the weak axiom of profit maximization (WAPM) be rejected for this firm?

2.2. Briefly explain why WAPM is only a necessary, and not sufficient, condition for profit maximization.

For the following two questions, consider that the firm has the following production function: $y = \min(1x_1, 2x_2)$. Moreover, consider that the price for inputs x_1 and x_2 are $w_1 = 1$ and $w_2 = 2$.

2.3. Carefully sketch the isoquant that allows the firm to produce exactly 10 units of output:

$$\{\, (x_1, x_2) \;\; in\ R_+^2 \;\mid\; \min(1x_1, 2x_2) = 10 \,\}$$

Carefully sketch into the same graph the isocost line that corresponds to the minimum costs c that allows the firm to produce those 10 units of output:

$$\{\, (x_1, x_2) \;\; in\ R_+^2 \;\mid\; 1x_1 + 2x_2 = c \,\}$$

Conclude from your graph what are the conditional factor demands for input 1 and 2 and what are the minimum costs c to produce 10 units of output.

2.4. Consider that the price for input 2 increases to 4, so that $w_2 = 4$. What are the new conditional factor demands for input 1 and 2 and what are the new minimum costs c to produce 10 units of output?

3. Constrained optimization (5 points)

A firm uses inputs x_1 and x_2 to produce output y. Imagine that the prices for x_1 and x_2 are $w_1 = 2$ and $w_2 = 1$ respectively. The production function is given by $y = x_1^{2/3} x_2^{1/3}$.

3.1. Calculate the minimum costs to produce $y = 10$.

3.2. What are the first-order and second-order condition for a constrained optimization problem as analysed in question 3.1? Briefly explain the intuition behind both conditions.

4. Cost functions (4 points)

4.1. Provide a rigorous (mathematical) proof for the following statement: with decreasing returns to scale the average cost must be upwards sloping.

4.2. Briefly explain when the short-run total costs are equal to the long-run total costs.