Universidade de Lisboa
ISEG – Lisbon School of Economics and Management
Undergraduate Degree in Finance
Subject: Financial Markets
Date: 04/12/2025
Time to complete the exam: 2:00 hours

A portfolio manager intends to build an efficient portfolio based on 2 different stocks, A and B, characterized by the following parameters, while the risk-free interest rate is represented by the 1-year spot rate.

			Covariances	
Assets	$E[R_i]$	σ_i	A	B
A	0,10	0,2		
B	0,20	0,5	0,07	
Market	0,15	0,3	0,03	0,04

1. Assuming that the following risk-free spot interest rates are known and the forward rate for a time to settlement equal to 9 months and a maturity equal to 3 months is 2,338% (with discrete compounding), compute the corresponding 1-year spot rate. **(2,5/20)**

Maturities (months)	Interest Rates (%)
3	2,069
6	2,117
9	2,162

2. Represent graphically the efficient frontier when short sales are not allowed (with a difference between two consecutive weights for each asset equal to 0,2), with the covariance mentioned and alternatively if they were fully independent, computing the minimum risk an investor could obtain from any portfolio combination with those 2 assets and using those 2 covariances. Explain the results obtained and their differences, as well as the impact of a lower covariance on diversification benefits. **(3,0/20)**
3. What would be the impact of allowing short sales on the efficient frontier and the maximum risk and risk, explaining adequately? **(1,5/20)**
4. What is the most efficient portfolio exclusively composed of these 2 assets, assuming that short sales and borrowing and lending at the risk-free rate are allowed (assume also that the risk-free rate corresponds to the answer to question 1, rounded to a percentage, or 2% if you haven't solved that question)? **(3,0/20)**
5. Represent graphically the efficient frontier for a portfolio that contains the risk-free asset and the portfolio resulting from the previous question, also quantifying its intercept and slope, as well as the segments of this frontier where the investor is lending and borrowing (if you haven't answered to the previous question, assume that the weights of assets A and B in that question are 80% and 20%). **(3,0/20)**
6. Comparing to question 4, what would be the impact on the optimization problem of disallowing short sales to compute the efficient portfolio and on the asset allocation of the portfolio composed by these 2 assets? **(1,5/20)**
7. Represent the security market line with the data provided, computing the market price of risk and the quantity of risk of each asset, as well as characterizing and explaining the level of risk of assets A and B (assuming that the risk-free rate is constant). **(3.0/20)**
8. Compare the risk-adjusted returns of both assets, using the Sharpe and the Treynor measures and characterizing the main differences between both measures and the results obtained. **(2,5/20)**

Enjoy your work!

Formulas

Forward rates:

$$ {}_{m}f_{n} = \left[\frac{(1+s_{m+n})^{m+n}}{(1+s_{m})^{m}} \right]^{\frac{1}{n}} - 1 $$

Expected return of a portfolio:

$$ \overline{R}_P = X_A \overline{R}_A + X_B \overline{R}_B $$

Standard-deviation of a portfolio:

$$ \sigma_P = \left[X_C^2 \sigma_C^2 + (1 - X_C)^2 \sigma_S^2 + 2 X_C (1 - X_C) \sigma_C \sigma_S \rho_{CS} \right]^{1/2} $$

Correlation coefficient:

$$ \rho_{ik} = \frac{\sigma_{ik}}{\sigma_i \sigma_k} $$

Weight of an asset to provide the minimum risk to a portfolio (for assets C and S):

$$ X_C = \frac{\sigma_S^2 - \sigma_C \sigma_S \rho_{CS}}{\sigma_C^2 + \sigma_S^2 - 2\sigma_C \sigma_S \rho_{CS}} $$

Expected return of a portfolio combining a risk-free asst and a portfolio A

$$ \overline{R}_C = R_F + \left(\frac{\overline{R}_A - R_F}{\sigma_A} \right) \sigma_C $$

Ratio between the excess return of a portfolio over a risk-free asset and the standard-deviation of that portfolio:

$$ \theta = \frac{\overline{R}_P - R_F}{\sigma_P} $$

Security market line:

$$ \overline{R}_i = R_F + \beta_i \left(\overline{R}_M - R_F \right) $$

Beta in a linear function:

$$ \beta_i = \frac{\sigma_{iM}}{\sigma_M^2} $$

Market price of risk:

$$ \frac{\overline{R}_M - R_F}{\sigma_M} $$

Quantity of risk:

$$ \frac{\sigma_{iM}}{\sigma_M} $$

Sharpe ratio:

$$ (\overline{R}_p - R_F)/\sigma_p $$

Treynor ratio:

$$ (\overline{\boldsymbol{R}}_{\boldsymbol{A}} - \boldsymbol{R}_{\boldsymbol{F}})/\boldsymbol{\beta}_{\boldsymbol{A}} $$