LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Mathematics Department
Undergraduate Degrees in Economics and Management
Mathematics 2

Normal Period Exam - January 9, 2018

Duration: 1h15

Name:	**Number:**

1. Consider the two variable function defined by

$$f(x,y) = 4x^2 + y^2.$$

a. **[1,0 points]** Sketch the level curves of f corresponding to the values $k = 1$, $k = 4$ and $k = 0$.

b. **[1,0 points]** Consider the ellipse $\mathcal{E}$ of equation $4x^2 + y^2 = 5$.

Compute all vectors $\vec{u}$ perpendicular to $\mathcal{E}$ at point $(1,1)$.

2. [2,0 points] Compute and classify the critical points of the function defined in $\mathbb{R}^2$ by

$$g(x, y) = x^3 + 3xy^2 - 15x - 12y.$$

3. **[2,0 points]** Find the maximum and the minimum of the function defined in $\mathbb{R}^2$ by

$$f(x, y) = x^2 y$$

over the set $M = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$.

4. [2,0 points] Using a double integral, compute the area of the region of the plane defined by

$$\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \, : \, y \geq -2 - \frac{1}{2}x \wedge y \leq 8 - \frac{1}{2}x^2\}.$$

5. [2,0 points] Compute the general solution of the differential equation

$$y''(x) - 4y'(x) + 4y(x) = 4x^2$$

[Sheet for complementary resolutions]