LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Mathematics Department
Undergraduate Degrees in Economics and Management

Repeat Period Exam - January 6th, 2019
Duration: 2h15m

Name: _____ **Student Number:** _____

1. Consider the matrix $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 4 & 0 \\ 2 & -2 & 1 \end{bmatrix}$.

a. (1,0 pts) Show that A admits one single real eigenvalue and compute it.

b. (1,0 pts) Determine the associated eigenspace.

2. Let B a invertible square matrix such that $B^2 + 3B$ is not invertible.

a. (0,5 pts) Show that $\lambda_1 = 0$ is not an eigenvalue of B.

b. (1,0 pts) Show that $\lambda_2 = -3$ is an eigenvalue of B.

3. (2,0 pts) For $\alpha \in \mathbb{R}$, $|\alpha| \neq \sqrt{2}$, consider the quadratic form

$$q(x, y, z) = z^2 + y^2 + \frac{1}{4}x^2 - azy + \frac{a}{2}zx.$$

Classify q in terms of the parameter a.

4. (1,5 pts) Consider the two variable function defined by the expression

$$f(x, y) = \ln((x^2 + x - 2)(y^2 - 3y + 2)).$$

Determine the domain D_f of f and represent it graphically. Is D_f an open set?

5. Consider the function f defined in $\mathbb{R}^2$ by

$$f(x,y) = \begin{cases} \dfrac{5yx^3}{y^2+6x^6} & \text{if } (x,y) \neq (0,0) \\ \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

a. (1,5 pts) Study the continuity of f at point $(0,0)$.

b. (1,5 pts) Compute $\dfrac{\partial f}{\partial x}(0,0)$, $\dfrac{\partial f}{\partial y}(0,0)$ if such quantities exist and determine for which vectors u the directional derivative $\partial_u f(0,0)$ exists.

6. (1,5 pts) Compute the coordinates of a vector orthogonal to the line of equation

$$x^3 + 3y^3 + e^{xy} = 4$$

at point $(0, 1)$. Hint: Interpret the line as a level curve.

7. (2,5 pts) Solve the initial value problem

$$\begin{cases} x^2 y' = x - y' \\ y(1) = 0 \end{cases}$$

8. (3,5 pts) Compute the absolute maximum and minimum of the function f defined by

$$f(x, y) = 2x^2 + y^2$$

over the set $A = \{(x, y) \in \mathbb{R}^2 : x^2 + 4y^2 \leq 1\}$.

9. (2,5 pts) Compute

$$\iint_A y dx dy$$

where $A = \{(x, y) \in \mathbb{R}^2 \, : \, x + y^2 \leq 0 \wedge x + y + 2 \geq 0\}$.

[Extra sheet for additional resolutions]

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