Lisbon School of Economics & Management
Universidade de Lisboa

Department of Mathematics
Undergraduate degrees in Economics, Finance and Management
Mathematics II

Regular Period Exam - June 5, 2023
Duration: 2h
Exam Notebook I

Name: _____ **Student Number:** _____

(1) Consider the matrix $A = \begin{bmatrix} a & 0 & 1 \\ 1 & 2 & 1 \\ a & 0 & 1 \end{bmatrix}$, $a \in \mathbb{R}$.

(a) (1.0 pts) Compute the eigenvalues of A depending on the parameter a.

(b) (1.5 pts) Assuming $a = 1$, determine if A is diagonalizable.

(2) Let B be a square matrix such that $B^3 = 0$.

(a) (1.5 pts) Show that 0 is an eigenvalue of B.

(b) (1.0 pts) Can we conclude that $B = 0$? Justify your answer.

(3) (2.0 pts) For $a \in \mathbb{R}$, consider the quadratic form

$$q(x, y, z) = y^2 + 2ayz + 2xy + x^2.$$

Classify q in terms of the parameter a.

(4) (2,0 pts) Consider the following function

$$f(x, y) = \frac{\ln(\sqrt{1 - x^2 - y^2})}{x^2 - y^2}.$$

Determine the domain D_f of f and represent it graphically. Is D_f a compact set? Justify your answer.

(5) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \dfrac{x \sin y}{\sqrt{x^2+y^2}} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

(a) (1.5 pts) Study the continuity of f at $(0,0)$.

(b) (1.5 pts) Compute $\dfrac{\partial f}{\partial x}(0,0)$ and $\dfrac{\partial f}{\partial y}(0,0)$ if such quantities exist. Is f differentiable at $(0,0)$? Justify your answer.

[Extra sheet for additional resolutions]

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(6) (2,0 pts) Let $w(x, y, z) = e^x y + \frac{z^2}{2}$ and

$$x(s,t) = \sin(s-t), \quad y(s,t) = s - t, \quad z(s,t) = \sqrt{s+t}.$$

Use the chain rule to show that

$$\frac{\partial w}{\partial s} + \frac{\partial w}{\partial t} = 1.$$

(7) (2.0 pts) Compute the optimal points of

$$f(x, y, z) = x^2 - 10y$$

over the set $D = \{(x, y, z) \in \mathbb{R}^3 \colon x^2 + y^2 + z^2 = 25\}$.

(8) (2.0 pts) Compute

$$\iint_A xe^{-2y}\,dxdy$$

where A is the region bounded by the axes of the plane and the curves $y = 1$ and $y = \ln x$.

(9) (2.0 pts) Solve the boundary value problem

$$\begin{cases} y'' + 4y = 0 \\ y(0) = 0,\ y(\frac{\pi}{4}) = 0 \end{cases}$$

[Extra sheet for additional resolutions]