Lisbon School of Economics & Management
Universidade de Lisboa

Department of Mathematics
Undergraduate degrees in Economics, Finance and Management
Mathematics II

Resit Period Exam - June 29, 2023
Duration: 2h
Exam Notebook I

Name: _____ **Student Number:** _____

(1) Consider the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$.

(a) (1.0 pts) Compute the eigenvalues of A.

(b) (1.5 pts) Determine if A is diagonalizable and compute the eigenspace associated to the smallest eigenvalue of A.

(2) Let B be a 3×3 matrix with eigenvalues equal to 0, 1 and 2.

(a) (1.5 pts) Determine the eigenvalues of $I + B$.

(b) (1.0 pts) Is $I + B$ an invertible matrix? Justify your answer.

(3) (2,0 pts) Consider the following function

$$f(x, y) = \frac{\sqrt{y - x^2}}{\ln(1 + x^2 + y^2 + \sqrt{1 - y})}$$

Determine the domain D_f of f and represent it graphically. Is D_f a compact set? Justify your answer.

(4) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^3 + y^4} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

(a) (1.5 pts) Study the continuity of f at $(0,0)$.

(b) (1.5 pts) Compute $\dfrac{\partial f}{\partial y}(0,0)$. Is f differentiable at $(0,0)$? Justify your answer.

[Extra sheet for additional resolutions]

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(5) (2.0 pts) For $|a| \neq 1$, consider the quadratic form

$$q(x, y, z) = x^2 + y^2 + z^2 + 2ayz.$$

Classify q in terms of the parameter a.

(6) (2,0 pts) Consider the function

$$f(x, y) = xe^{-x}(y^2 - 4y)$$

Determine and classify the critical points of f.

(7) (2.0 pts) Compute the optimal points of

$$f(x, y, z) = x + y + z$$

over the set $D = \{(x, y, z) \in \mathbb{R}^3 \colon x^2 + y^2 + z^2 = 2\}$.

(8) (2.0 pts) Use the double integral to calculate the area of the planar region

$$A = \left\{(x, y) \in \mathbb{R}^2 \colon y^2 \leq x \leq 4,\, 0 \leq xy \leq 1\right\}.$$

(9) (2.0 pts) Solve the initial value problem

$$\begin{cases} xy' + y + x = 0 \\ y(1) = 0 \end{cases}$$

[Extra sheet for additional resolutions]