GENERAL EQUILIBRIUM

(Lectures 7 and 8)

Exercise 48

There are 300 consumers and 2 commodities in an economy. Consumer *i* has a utility function given by $u_i(x_i^1, x_i^2) = \ln(x_i^1) + \ln(x_i^2)$.

Find the equilibrium prices knowing that in a Walras-equilibrium consumer *i* obtains the bundle (10,5).

Exercise 49

Consider an exchange economy with 2 consumers. Consumer 1's utility function is given by $u_1(x_1^1, x_1^2) = (1 + x_1^2)e^{x_1^1}$ and her endowment is $w_1 = (2,1)$. Consumer 2's utility function is $u_2(x_2^1, x_2^2) = x_2^1 x_2^2$ and endowment is $w_2 = (2,3)$. Determine:

1. the individual demand functions;
2. the excess demand function;
3. the Walrasian equilibrium;
4. the contract curve;
5. the core.

Exercise 50

Consider the classic Robinson Crusoe economy, where all production and consumption are carried out by a single agent. Robinson is endowed with $\bar{L}$ man-hours per week. On his island there is only one productive activity, oyster harvesting from an oyster bed, and only one input to production, Robinson's labour (L). Robinson derives utility from consuming oysters (c) and leisure (R). Note that we must have R=$\bar{L}$-L.

1. Let x=F(L) denote the production function for oysters, where L is the amount of labour used. Assume F'>0, F''<0, and F'(0)=+∝. Let u(c, R) be Robinson's utility function, with

$$\frac{\partial u}{\partial c} > 0, \frac{\partial u}{\partial R} > 0, \frac{\partial^2 u}{\partial R^2} < 0, \frac{\partial^2 u}{\partial c^2} < 0, \frac{\partial^2 u}{\partial c \partial R} > 0.$$

a. Determine the efficient allocation in this economy both analytically and graphically;

b. Let market prices for output and labour be $p>0$ and $w>0$, respectively. Find the general equilibrium.

2. Now asssume $F(L)=L^{\alpha}$, where $0<\alpha<1$, and u(c, ~~R~~)= $c^{\beta} R^{1-\beta}$, where $0<\beta<1$, and find the general equilibrium.

Exercise 51

Consider a one-consumer, one-producer economy. Compute the equilibrium prices, consumptions, and profits when the consumer's utility function is given by $u(x,y)=\ln(x)+\ln(y)$ - where y denotes leisure - the production function is $f(z)=\sqrt{z}$ - where z denotes labour - and the total endowment of time is 1.

Exercise 52

Consider an economy with 2 commodities and 2 consumers, whose utility functions are given by $u_i(x_i^1,x_i^2)=\sqrt{x_i^1 x_i^2}$, $i=1,2$ and endowments are $w_1=(1,3)$ and $w_2=(3,1)$. Determine the core of this economy.

Exercise 53

Consider a three-agent economy where each agent has a utility function $u(x,y)=xy$ and endowments are $w_1=w_2=(1,14)$ and $w_3=(27,1)$. Show that the following allocation belongs to the core: $(x_1,y_1)=(6,6)$, $(x_2,y_2)=(7,7)$, and $(x_3,y_3)=(16,16)$.

Exercise 54

V, Ex. 18.2, p.357.