Lecture 6

Varian, Chs. 9, 10, and 17.1 to 17.5; MWG, Chs. 3.I and 4.B

1 Summary of Lectures 1, 2, and 3: Production theory

2 Summary of Lectures 4 and 5: Consumption theory

2.1 Preference orders

2.2 The utility function

2.3 The utility maximization problem

2.4 The expenditure minimization problem

2.5 Duality in consumption

3 Choice

3.1 Comparative statics of consumer behavior

3.2 Income and substitution effects

3.2.1 The Slutsky equation

Remark 1 *Integrability: if a set of demand functions give rise to symmetric and negative semi-definite matrix of substitution terms then we can solve for the indirect utility function and the expenditure function. (c.f. the condition determining whether we can go from conditional demand functions to the technology).*

3.3 Revealed preference

4 Consumers' surplus

4.1 Measuring welfare effects

4.1.1 The compensating variation (CV)

In general a policy change may affect both income and prices. Given that a change takes place what income compensation is required to leave the consumer as well of as before the change.

$$CV = m^1 - e(p^1, u^0) = \mu(p^1; p^1, m^1) - \mu(p^1; p^1, m^0)$$

where $\mu(q;p,m) = e(q, v(p,m))$. Suppose only one price changes and income remains constant, $m^0 = m^1$. Specifically, let p^1 fall from p_1^0 to p_1^1. In this case,

$$e(p^0,u^0) - e(p^1,u^0) = \int_{p_1^1}^{p_1^0} \frac{\partial e}{\partial p_1} dp_1 = \int_{p_1^1}^{p_1^0} h_1(p,u^0) dp_1.$$

4.1.2 The equivalent variation (EV)

Suppose prices and income remain the same. What income change would be necessary to give the consumer the same utility as he would have obtained if the price change from p_1^0 to p_1^1 had taken place?

By the same argument as above we can obtain:

$$EV = e(p^0,u^1) - e(p^1,u^1) = \int_{p_1^1}^{p_1^0} h_1(p,u^1) dp_1.$$

Note that the consumer surplus, CS, obeys EV > CS > CV.

4.1.3 Quasi-linear utility and no income effects

No income effects means that the consumption of the good depends only on the relative prices and not on income (provided that the income suffices to finance the desired quantity). Consequently the Hicksian demand curves and the Marshallian demand curve coincide and CV must equal EV.

5 Demand

5.1 Endowments in the budget constraint

The UMP becomes:

$$\begin{aligned} & \underset{x}{Max}\ u(x) \\ s.t.\ p \cdot x &= p \cdot \omega \end{aligned}$$

to obtain a demand function $x(p, p \cdot \omega)$.

The Slutsky equation becomes:

$$\frac{\partial x_j(p, p\cdot\omega)}{\partial p_i} = \frac{\partial h_j(p,u)}{\partial p_i} - \frac{\partial x_j(p,p\cdot\omega)}{\partial m}(\omega_i - x_i).$$

Now the income effect depends on the net demand for good i.

In the case of a normal good, when the price goes up, both substitution and income effect push towards a reduced consumption of the good. But if the consumer is a net supplier of this good, his income increases and the additional endowment income effect may actually increase the consumption of the good.

5.2 Homothetic utility

In utility theory, a homothetic utility function $\approx$ a homogenous of degree 1 utility function.

If preferences are homothetic, a proportional increase in the consumption of all goods then yields a proportional increase in utility. For given prices the same consumption mix is optimal regardless of income. Hence, the expenditure function can be expressed as $e(p,u) = e(p)u$ implying that $v(p,m) = v(p)m$ and (by Roy's identity) $x_i(p,m) = x_i(p)m$, i.e., the demand functions are linear functions of income.

5.3 Aggregation across consumers

Let $x_i^k(p, m_i)$ denote the demand function of consumer i for good k and assume that there are n consumers. The aggregate demand function is $X(p, m_{1,...,} m_n) = \sum\limits_{i=1}^{n} x_i(p, m_i)$ and the aggregate demand for good j is $X^j(p, m)$, where m is the vector of incomes.

If individual demands are continuous, the agrregate demand function is continuous - continuity of the individual demands is sufficient but not necessary.

Aggregate demand is a function of price and aggregate income if agents have *Gorman-type utility functions*: $v_i(p, m_i) = a_i(p) + b(p)m_i$ - Gorman-type utility functions are sufficient and necessary conditions for the representative consumer model to hold.

The crucial feature is that changes in income affects all consumers' behavior the same way. Therefore demand only depends on the aggregate income and not on how it is distributed among individuals. Homothetic utility functions ($v(p,m) = v(p)m$) and quasilinear utility functions ($v(p,m) = v(p) + m$) have this property.

5.4 Strictly convex preferences ensure continuity...

6 Exchange

6.1 Walrasian equilibrium

The solution to a consumer's utility maximization problem

$$\begin{aligned} & \underset{x_i}{Max}\ u(x_i) \\ s.t.\ p \cdot x_i &= p \cdot \omega_i \end{aligned}$$

when taking prices as given is the consumer's demand function. In equilibrium aggregate demand cannot exceed endowments; thus, a Walrasian equilibrium is a pair (p^*, x^*) such that:

$$\sum_i x_i(p^*, p^* \cdot \omega_i) \leq \sum_i \omega_i.$$

If all goods are "desirable", demand equals suppply in all markets.

The aggregate excess demand function is:

$$z(p) = \sum_i [x_i(p, p \cdot \omega_i) - \omega_i].$$

6.2 Edgeworth box (2 consumer case)

6.3 Existence of Walrasian equilibria

$z(p)$ satisfies:

1. homogeneity of degree zero in prices
2. continuity (when all individual demand functions are continuous)
3. Walra's law: for any p, $p \cdot z(p) \equiv 0$.

Proof. Since all consumers are on their budget constraints in optimum the value of their endowments must equal the value of the demanded bundles. Aggregation across consumers preserves this property. ■

Proposition 2 *If we know that all markets but market k clears and $p_k > 0$, then market k must also clear.*

Proposition 3 *If a good is in excess supply in a Walrasian equilibrium, i.e., $z_j(p^*) < 0$, it must be a free good: $p_j^* = 0$.*

Proof. If not then $p_j \cdot z_j(p) < 0$ implying that $p \cdot z(p) < 0$ (since the excess demand for each good in a Walrasian equilibrium is non-positive and prices are positive). This contradicts Walras' law. ■

If we assume that all good are desirable so that $p_i = 0$ implies $z_i(p) > 0$, then the excess demand must be equal to zero for each good.

Proposition 4 *If all goods are desirable and p^* is a Walrasian equilibrium, then $z^*(p) = 0$.*

Definition 5 *Walrasian equilibrium: (x^*, p^*) is a Walrasian equilibrium iff (i) the allocation is feasible $\sum_i x_i^* = \sum_i \omega_i$ and (ii) each agent makes an optimal choice: if x_i' is preferred to x_i, then $px_i' > p\omega_i$.*

When all goods are desirable a Walrasian equilibrium can be defined as a (x^*, p^*) such that sum of the normalized prices equals 1. All possible prices can now be represented as points on a unit simplex (with a dimension equal to the number of prices minus one). By construction it is a compact set.

Is there a price vector p^* such that excess demand is zero?

Proposition 6 *If $z : S^{k-1} \to \Re^k$ is a continuous function that satisfies Walras' law, $pz(p) \equiv 0$, then there exists some p^* such that $z(p^*) \leq 0$.*

For any given p the consumers' choices result in some $z(p)$. If there is excess demand (supply) for a good this would tend to increase (reduce) its price. This describes a relationship between prices and new adjusted prices. We can use this reasoning to prove the existence of an equilibrium. Specifically, if the price adjustment is a continuous function, say $g(p)$, from the price simplex to itself Brouwer's fixed point theorem ensures that there exists a p such that $g(p) = p$.

We can construct g so that this can only happen when excess demand is zero. Let

$$g_i(p) = \frac{p_i + \max(0, z_i(p))}{1 + \sum_{j=1}^{k} \max(0, z_j(p))} \text{ for } i = 1, ..., k.$$

which is continuous in prices and maps all price vectors back into the price simplex. Consequently, there exists a p such that $z(p) = 0$.

The key requirement for existence of a Walrasian equilibrium is continuity of the aggregate excess demand function. This is the case if

consumer preferences are convex or, if there are infinitely many consumers so that each consumer is small compared with the size of the

market. The assumption about competitive behavior is more plausible if there are many small consumers. Thus, when competitive behavior

seems reasonable so does equilibrium analysis.