ANÁLISE MATEMÁTICA I

Ficha Nº14

1. Calcule os seguintes limites

   (a) $\lim\limits_{x\to+\infty} \frac{1}{x}\int_1^x \frac{\ln t}{1+t}dt$;

   (b) $\lim\limits_{x\to 0} \frac{x^2}{1-e^{x^2}}\int_0^x e^{t^2}dt$.

2. Calcule os seguintes integrais:

   (a) $\int_{-\infty}^{+\infty} \frac{1}{x^2+4x+9}dx$;

   (b) $\int_0^{\frac{1}{2}} \frac{1}{x\ln x}dx$;

   (c) $\int_0^{\frac{1}{2}} \frac{1}{x\ln^2 x}dx$;

   (d) $\int_2^{+\infty} \frac{1}{x\ln x}dx$;

   (e) $\int_2^{+\infty} \frac{1}{x\ln^2 x}dx$;

   (f) $\int_0^{\frac{\pi}{2}} \text{cotg}x dx$;

   (g) $\int_0^{+\infty} \frac{\text{arctg}x}{1+x^2}dx$;

3. Determine a natureza dos integrais:

   (a) $\int_0^{+\infty} \sin 3x dx$;

   (b) $\int_2^{+\infty} \frac{1}{x^2-3}dx$;

   (c) $\int_2^{+\infty} \frac{1}{x^2-4}dx$;

   (d) $\int_0^{+\infty} \frac{e^{-x}}{x^\alpha}dx$;

   (e) $\int_0^{+\infty} \frac{1}{x^\alpha+x^\beta}dx$;

   (f) $\int_0^{+\infty} \frac{1}{\sqrt[3]{x}+2\sqrt[4]{x}+x^2}dx$;

   (g) $\int_1^{+\infty} \frac{1}{2x+\sqrt[3]{x^2+1}+5}dx$;

   (h) $\int_{-\infty}^{+\infty} \frac{1}{x^2+\sqrt[3]{x^4+1}}dx$;

   (i) $\int_0^{+\infty} \frac{x}{\sqrt{x^5+1}}dx$;

   (j) $\int_{-\infty}^{+\infty} \frac{1}{\sqrt[3]{1-x^4}}dx$;

   (k) $\int_{0,2} \frac{1}{\ln x}dx$;

   (l) $\int_{\frac{\pi}{2}}^{+\infty} \frac{\sin x}{x^\alpha}dx$

4. A função Gamma de Euler é definida pela expressão

$$\Gamma(t) = \int_0^{+\infty} x^{t-1} e^{-x} dx.$$

Prove que $\Gamma(t)$ está definida para todo $t \in]0, +\infty[$.