(Note: Justify all your answers and present the calculations)

1. Consider the following output, from the Solver/Excel solution of the LP problem formulated, to determine the tons of each of the three products ( $\mathbf{P 1}, \mathbf{P} 2$ e P3) that maximize the total profit:

$$
\begin{aligned}
& \text { Max } Z=5 x_{1}+8 x_{2}+10 x_{3} \\
& \text { s. to: }\left\{\begin{array}{lr}
x_{1}+2 x_{2}+x_{3} \leq 2000 & \text { machine (capacity in } m . h \text {.) } \\
x_{1}+0,5 x_{2}+2 x_{3} \leq 4000 & \text { waehouse (capacity in } m^{3} \text { ) } \\
x_{1}+x_{2} \quad \geq 300 & \text { selling agreement (ton.) } \\
x_{1}, x_{2}, x_{3} \geq 0 &
\end{array}\right.
\end{aligned}
$$

## Microsoft Excel 12.0 Sensitivity Report

## Adjustable Cells

| Cell | Name | Final <br> Value | Reduced <br> Cost | Objective <br> Coefficient | Allowable <br> Increase | Allowable <br> Decrease |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\$ C \$ 7$ | P 1 | 300 | 0 | 5 | 5 | 7 |
| $\$ \mathrm{D} \$ 7$ | P 2 | 0 | -7 | 8 | 7 | $1 \mathrm{E}+30$ |
| $\$ \mathrm{E} \$ 7$ | P 3 | 1700 | 0 | 10 | $1 \mathrm{E}+30$ | 5 |

Constraints

| Cell | Name | Final <br> Value | Shadow <br> Price | Constraint <br> R.H. Side | Allowable <br> Increase | Allowable <br> Decrease |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| $\$ F \$ 3$ | machine $($ m.h. $)$ | 2000 | 10 | 2000 | 150 | 1700 |
| $\$ F \$ 4$ | warehouse $\left(\mathrm{m}^{3}\right)$ | 3700 | 0 | 4000 | $1 \mathrm{E}+30$ | 300 |
| $\$ F \$ 5$ | selling agreement (ton.) | 300 | -5 | 300 | 1700 | 300 |

a) (3 points) Write and explain the above results, including decision variables, slack variables and shadow prices.
b) ( 1,5 points) Identify and justify a value for the unitary profit of $\mathbf{P} 3$ that will induce changes on the optimal solution plan.
c) (2 points) The board of directors is considering the replacement of the machine by an identical one with an increased capacity. Justify what the maximum capacity of the new machine should be in order to keep the production of the same products. Compute the change in the optimal total profit if the a machine with that maximum capacity costs 50 m.u.
d) ( 1,5 points) Suppose that a flood reduces the warehouse capacity to half. Explain how should the initial formulation be changed in order to obtain a model that will help you decide to either: i) keep the old warehouse, in the conditions after the flood or, ii) rent another warehouse with a capacity of 3500 $m^{3}$ at a cost of 2000 m.u..
2. David, a student living in Lisboa, usually takes the underground. The relevant underground network follows in figures $\mathbf{A}$ and $\mathbf{B}$.


Figure A


Figure B
a) (3 points) Suppose that a computer network is to be set connecting all the different underground stations. With that purpose, a cable network will be installed along the underground tunnels, thus linking the computers together. Every station has one computer and each needs to get connected to the network. Considering figure $\mathbf{A}$ where numbers on the links represent the cost (in m.u.) between the stations, determine the solution that minimizes the total cost of cables to be installed.
b) ( 3,5 points) Consider that the numbers on arcs in $\mathbf{B}$ represent the time (in minutes) between stations. With the purpose of moving as fast as possible from M.Pombal to Oriente, David chose the best direction for every link but for the one between Aeroporto and Alameda, as depicted in B. Complete the Excel worksheet and the Solver window in the attachment, while writing all parameters and formulas needed by David to solve the problem.
3. Consider the following simplex tableau from an LP maximization problem with two decision variables and three functional $\leq$ constraints:

| VB | Z | $x_{1}$ | $x_{2}$ | $x_{3}$ | $x_{4}$ | $x_{5}$ | R.H.S. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Z | 1 | -2 | 0 | 0 | 1 | 0 | 10 |
| - | 0 | -1 | 0 | 1 | $-\cdots$ | $-\cdots$ | $-\cdots$ |
|  | 0 | 0 | 1 | 0 | 3 | 0 | 9 |
|  | 0 | -2 | 0 | 0 | -1 | 1 | 4 |

a) (1 points) Write and classify the primal solution shown on the tableau.
b) (1,5 points) Indicate the consequences to the solution shown on the tableau of an increment by one unit in the value of $x_{4}$.
c) (1,5 points) Solve the problem.
d) (1,5 points) Classify and explain the sentence: "The feasible region of the above problem is convex and bounded".


