INSTITUTO SUPERIOR DE ECONOMIA E GESTÃO

OPERATIONS RESEARCH

2010/01/29 "**Época Recurso"** Duration: **2h**

(**Note**: Justify all the answers and present the calculations.)

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1. An individual, among many others, strongly invested in real estate based funds, and finally managed to recover 25 thousand *m.u.* (monetary units) which he intends to invest during a certain period of time. After the past experience, his goal is to minimize the risk, however he would like to achieve a minimum return of 2 thousand *m.u.* at the end of the time period. The characteristics of the financial products, which he ponders to include in his portfolio, made him formulate such LP problem, where x_i represents the amount (in 10^3 *m.u.*) to be invested on product i=1,2:

$$Min\, z = x_1 + 2x_2$$

$$\text{s.to} \begin{cases} x_1 + x_2 \leq 25 & \text{(budget constraint)} \\ 0{,}5x_1 + 0{,}8x_2 \geq 2 & \text{(return constraint)} \\ x_1,\ x_2 \geq 0 \end{cases}$$

a) (3 points) Solve graphically the given problem. Present and interpret the optimum value of the decision and slack (auxiliary) variables.

b) (2 points) Write the dual and determine its optimal solution (only the decision variables). Note that you can take advantage on the solution and resolution of a).

2. An account manager criticized the approach of the above problem, arguing that this way the return achieved would never be higher than the minimum required. According to him, the objective function should translate the maximization of the return and the risk might be controlled by constraints imposed on the portfolio composition. Besides, he suggested two additional financial products to take into account. Using the OR knowledge he formulated the following problem which he solved through S*olver/Excel* (Appendices A).

	X1	X2	X3	X4	total		Right Hand Sides (thousand *m.u.*)
Budget	1	1	1	1	25	<=	25
Risk 1	1	1	0	0	10	<=	15
Risk 2	0	0	1	1	15	<=	15
Risk 3	1	0	1	0	15	>=	15
Return	0,50	0,80	0,75	0,90	19,25		
Solution	0	10	15	0			

Taking into account the elements of Appendices A help clarify the following doubts.

a) (2 points) How much should be invested on each product and what is the associated total return?

b) (2 points) How much changes the total return, if Risk 2 constraint is changed allowing a maximum of 14 thousand *m.u.* to invest, instead of the current 15 thousand *m.u.*?

c) (2 points) Could you quantify the change in the total return, if it is required that the total invested in products 3 and 4 (RHS of constraint Risk 2) does not surpass the 9 thousand *m.u.*?

d) (2 points) How much does the total return change, if the return of product 1 increases from 0,5 to 0,6? Identify the optimal solution for this situation.

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3. From the present crisis, a consensus was reached for the needs to enhance banking supervision. In this sense, it was decided to reinforce the communication network of several services A, B, C, D, E and F. With such goal, information, displayed on the next table, was gathered on the costs of connection (in *m.u.*) between the pairs of services so that a set of connections of minimum total cost would be selected, which assures communication among the 6 services.

	A	B	C	D	E	F
A		-	-	3	1	6
B	-		3	-	-	3
C	-	3		5	3	4
D	3	-	5		2	5
E	1	-	3	2		-
F	6	3	4	5	-	

(-) unfeasible connection.

a) (1 point) Identify the network optimization problem at stake.

b) (3 points) Apply an algorithm studied to identify the services that should be directly connected and the total cost involved.

4. (3 points) A family decided to reorganize its budget management, after reaching the conclusion that their financial means were not enough to cover their life style expenses (monthly expenses in value of 3000 *m.u.*). The father has a net monthly wage of 1200 *m.u.* and the mother 1300 *m.u.*, and the son usually contributes with 100 *m.u. per* month. On the total outcome, 2000 *m.u.* are considered essential expenses. The expenses that can be cut down, or even suppressed, fall into 3 categories: meals at restaurants (300 *m.u. per* month), trips on the weekends (500 *m.u. per* month) and concerts (200 *m.u. per* month). Each member of the family was asked to quantify, on a scale from a minimum of 1 to a maximum of 5, what satisfaction they would grant to 1 *m.u.* spent on each of the four features, on which the following table results:

	Essentials	Restaurants	Trips	Concerts
Father	5	3	2	1
Mother	5	1	2	3
Son	1	4	5	4

It is intended to assign the responsibility of the payment of the expenses to each member of the family, as well as to decide which expenses to cut down or suppress, in such way that total satisfaction is maximized.

Knowing that this problem is a variant of the transportation problem present its LP formulation.