

# **GESTÃO FINANCEIRA II**

## **PROBLEM SET 2 - SOLUTIONS**

**(FROM BERK AND DEMARZO'S "CORPORATE FINANCE")**

**LICENCIATURA – UNDERGRADUATE COURSE**

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## Chapter 8

# Valuing Bonds

8-3. The following table summarizes prices of various default-free, zero-coupon bonds (expressed as a percentage of face value):

Maturity (years)	1	2	3	4	5
Price (per \$100 face value)	\$95.51	\$91.05	\$86.38	\$81.65	\$76.51

- Compute the yield to maturity for each bond.
- Plot the zero-coupon yield curve (for the first five years).
- Is the yield curve upward sloping, downward sloping, or flat?
  - Use the following equation.

$$1 + \text{YTM}_n = \left( \frac{\text{FV}_n}{P} \right)^{1/n}$$

$$1 + \text{YTM}_1 = \left( \frac{100}{95.51} \right)^{1/1} \Rightarrow \text{YTM}_1 = 4.70\%$$

$$1 + \text{YTM}_2 = \left( \frac{100}{91.05} \right)^{1/2} \Rightarrow \text{YTM}_2 = 4.80\%$$

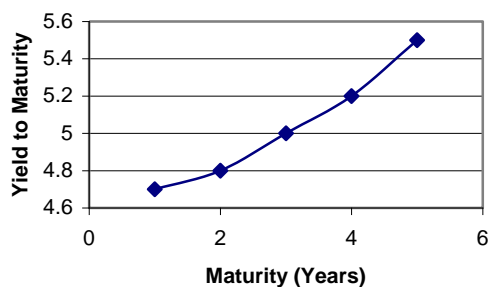
$$1 + \text{YTM}_3 = \left( \frac{100}{86.38} \right)^{1/3} \Rightarrow \text{YTM}_3 = 5.00\%$$

$$1 + \text{YTM}_4 = \left( \frac{100}{81.65} \right)^{1/4} \Rightarrow \text{YTM}_4 = 5.20\%$$

$$1 + \text{YTM}_5 = \left( \frac{100}{76.51} \right)^{1/5} \Rightarrow \text{YTM}_5 = 5.50\%$$

- The yield curve is as shown below.

**Zero Coupon Yield Curve**



- The yield curve is upward sloping.

- 8-5.** In the box in Section 8.1, Bloomberg.com reported that the three-month Treasury bill sold for a price of \$100.002556 per \$100 face value. What is the yield to maturity of this bond, expressed as an EAR?

$$\left(\frac{100}{100.002556}\right)^4 - 1 = -0.01022\%$$

- 8-14.** Suppose you purchase a 30-year, zero-coupon bond with a yield to maturity of 6%. You hold the bond for five years before selling it.

- If the bond's yield to maturity is 6% when you sell it, what is the internal rate of return of your investment?
- If the bond's yield to maturity is 7% when you sell it, what is the internal rate of return of your investment?
- If the bond's yield to maturity is 5% when you sell it, what is the internal rate of return of your investment?
- Even if a bond has no chance of default, is your investment risk free if you plan to sell it before it matures? Explain.

- Purchase price =  $100 / 1.06^{30} = 17.41$ . Sale price =  $100 / 1.06^{25} = 23.30$ . Return =  $(23.30 / 17.41)^{1/5} - 1 = 6.00\%$ . I.e., since YTM is the same at purchase and sale, IRR = YTM.
- Purchase price =  $100 / 1.06^{30} = 17.41$ . Sale price =  $100 / 1.07^{25} = 18.42$ . Return =  $(18.42 / 17.41)^{1/5} - 1 = 1.13\%$ . I.e., since YTM rises, IRR < initial YTM.
- Purchase price =  $100 / 1.06^{30} = 17.41$ . Sale price =  $100 / 1.05^{25} = 29.53$ . Return =  $(29.53 / 17.41)^{1/5} - 1 = 11.15\%$ . I.e., since YTM falls, IRR > initial YTM.
- Even without default, if you sell prior to maturity, you are exposed to the risk that the YTM may change.

- 8-19.** What is the price of a three-year, default-free security with a face value of \$1000 and an annual coupon rate of 4%? What is the yield to maturity for this bond?

The price of the bond is

$$P = \frac{CPN}{1+YTM_1} + \frac{CPN}{(1+YTM_2)^2} + \dots + \frac{CPN + FV}{(1+YTM_N)^N} = \frac{40}{(1+.04)} + \frac{40}{(1+.04)^2} + \frac{40+1000}{(1+.04)^3} = \$986.58.$$

The yield to maturity is

$$P = \frac{CPN}{1+YTM} + \frac{CPN}{(1+YTM)^2} + \dots + \frac{CPN + FV}{(1+YTM)^N}$$

$$\$986.58 = \frac{40}{(1+YTM)} + \frac{40}{(1+YTM)^2} + \frac{40+1000}{(1+YTM)^3} \Rightarrow YTM = 4.488\%$$

**8-20. What is the maturity of a default-free security with annual coupon payments and a yield to maturity of 4%? Why?**

The maturity must be one year. If the maturity were longer than one year, there would be an arbitrage opportunity.

**8-23. Prices of zero-coupon, default-free securities with face values of \$1000 are summarized in the following table:**

Maturity (years)	1	2	3
Price (per \$1000 face value)	\$970.87	\$938.95	\$904.56

**Suppose you observe that a three-year, default-free security with an annual coupon rate of 10% and a face value of \$1000 has a price today of \$1183.50. Is there an arbitrage opportunity? If so, show specifically how you would take advantage of this opportunity. If not, why not?**

First, figure out if the price of the coupon bond is consistent with the zero coupon yields implied by the other securities.

$$970.87 = \frac{1000}{(1+YTM_1)} \rightarrow YTM_1 = 3.0\%$$

$$938.95 = \frac{1000}{(1+YTM_2)^2} \rightarrow YTM_2 = 3.2\%$$

$$904.56 = \frac{1000}{(1+YTM_3)^3} \rightarrow YTM_3 = 3.4\%$$

According to these zero coupon yields, the price of the coupon bond should be:

$$\frac{100}{(1+.03)} + \frac{100}{(1+.032)^2} + \frac{100+1000}{(1+.034)^3} = \$1186.00.$$

The price of the coupon bond is too low, so there is an arbitrage opportunity. To take advantage of it:

	Today	1 Year	2 Years	3 Years
Buy 10 Coupon Bonds	11835.00	+1000	+1000	+11,000
Short Sell 1 One-Year Zero	+970.87	1000		
Short Sell 1 Two-Year Zero	+938.95		1000	
Short Sell 11 Three-Year Zeros	+9950.16			11,000
Net Cash Flow	24.98	0	0	0

**8-24. Assume there are four default-free bonds with the following prices and future cash flows:**

Bond	Price Today	Cash Flows		
		Year 1	Year 2	Year 3
A	\$934.58	1000	0	0
B	881.66	0	1000	0
C	1,118.21	100	100	1100
D	839.62	0	0	1000

**Do these bonds present an arbitrage opportunity? If so, how would you take advantage of this opportunity? If not, why not?**

To determine whether these bonds present an arbitrage opportunity, check whether the pricing is internally consistent. Calculate the spot rates implied by Bonds A, B, and D (the zero coupon bonds), and use this to check Bond C. (You may alternatively compute the spot rates from Bonds A, B, and C, and check Bond D, or some other combination.)

$$934.58 = \frac{1000}{(1 + YTM_1)} \Rightarrow YTM_1 = 7.0\%$$

$$881.66 = \frac{1000}{(1 + YTM_2)^2} \Rightarrow YTM_2 = 6.5\%$$

$$839.62 = \frac{1000}{(1 + YTM_3)^3} \Rightarrow YTM_3 = 6.0\%$$

Given the spot rates implied by Bonds A, B, and D, the price of Bond C should be \$1,105.21. Its price really is \$1,118.21, so it is overpriced by \$13 per bond. Yes, there is an arbitrage opportunity.

To take advantage of this opportunity, you want to (short) Sell Bond C (since it is overpriced). To match future cash flows, one strategy is to sell 10 Bond Cs (it is not the only effective strategy; any multiple of this strategy is also arbitrage). This complete strategy is summarized in the table below.

	Today	Year	Years	Years
d C	11,182.10	-1,000	-1,000	-11,000
d A	-934.58	1,000	0	0
d B	-881.66	0	1,000	0
Bond D	-9,235.82	0	0	11,000
Net Cash Flow	130.04	0	0	0

Notice that your arbitrage profit equals 10 times the mispricing on each bond (subject to rounding error).

**8-25. Suppose you are given the following information about the default-free, coupon-paying yield curve:**

Maturity (years)	1	2	3	4
Coupon rate (annual payments)	0.00%	10.00%	6.00%	12.00%
YTM	2.000%	3.908%	5.840%	5.783%

- Use arbitrage to determine the yield to maturity of a two-year, zero-coupon bond.
- What is the zero-coupon yield curve for years 1 through 4?

- We can construct a two-year zero coupon bond using the one and two-year coupon bonds as follows.

	Cash Flow in Year:			
	1	2	3	4
1-year coupon bond (\$1000 Face Value)	100	1,100		
1-year zero (\$100 Face Value)	100			
2-year zero (\$1100 Face Value)	-	1,100		

$$\text{Now, Price(2-year coupon bond)} = \frac{100}{1.03908} + \frac{1100}{1.03908^2} = \$1115.05$$

$$\text{Price(1-year bond)} = \frac{100}{1.02} = \$98.04.$$

By the Law of One Price:

$$\begin{aligned} \text{Price(2 year zero)} &= \text{Price(2 year coupon bond)} - \text{Price(One-year bond)} \\ &= 1115.05 - 98.04 = \$1017.01 \end{aligned}$$

Given this price per \$1100 face value, the YTM for the 2-year zero is (Eq. 8.3)

$$YTM(2) = \left( \frac{1100}{1017.01} \right)^{1/2} - 1 = 4.000\%.$$

b. We already know  $YTM(1) = 2\%$ ,  $YTM(2) = 4\%$ . We can construct a 3-year zero as follows:

	Cash Flow in Year:			
	1	2	3	4
1-year coupon bond (\$1000 face value)	60	60	1,060	
2-year zero (\$60 face value)	(60)			
1-year zero (\$60 face value)	-	(60)		
3-year zero (\$1060 face value)	-	-	1,060	

$$\text{Now, Price(3-year coupon bond)} = \frac{60}{1.0584} + \frac{60}{1.0584^2} + \frac{1060}{1.0584^3} = \$1004.29.$$

By the Law of One Price:

$$\begin{aligned} \text{Price(3-year zero)} &= \text{Price(3-year coupon bond)} - \text{Price(One-year zero)} - \text{Price(Two-year zero)} \\ &= \text{Price(3-year coupon bond)} - \text{PV(coupons in years 1 and 2)} \\ &= 1004.29 - 60 / 1.02 - 60 / 1.04^2 = \$889.99. \end{aligned}$$

Solving for the YTM:

$$YTM(3) = \left( \frac{1060}{889.99} \right)^{1/3} - 1 = 6.000\%.$$

Finally, we can do the same for the 4-year zero:

	Cash Flow in Year:			
	1	2	3	4
1-year coupon bond (\$1000 face value)	120	120	120	1,120
1-year zero (\$120 face value)	(120)			
1-year zero (\$120 face value)	—	(120)		
2-year zero (\$120 face value)	—	—	(120)	
4-year zero (\$1120 face value)	—	—	—	1,120

$$\text{Now, Price(4-year coupon bond)} = \frac{120}{1.05783} + \frac{120}{1.05783^2} + \frac{120}{1.05783^3} + \frac{1120}{1.05783^4} = \$1216.50.$$

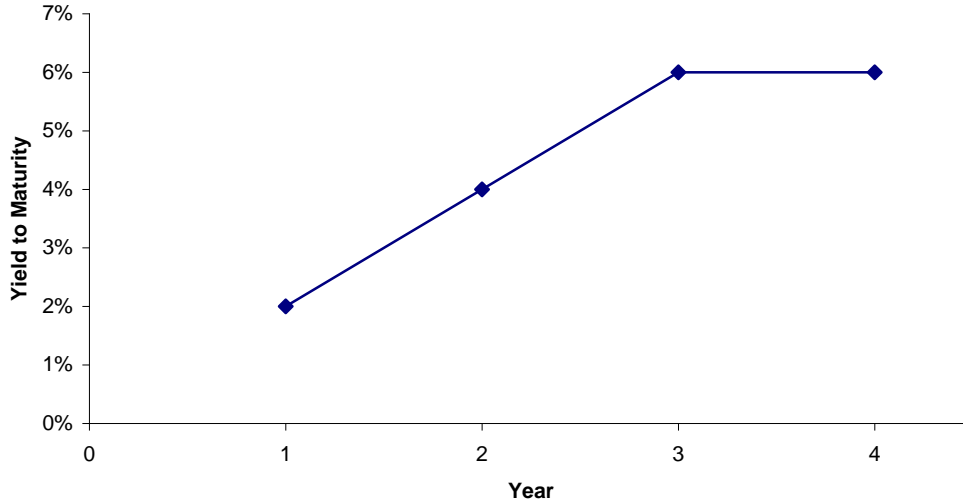
By the Law of One Price:

$$\begin{aligned} \text{Price(4-year zero)} &= \text{Price(4-year coupon bond)} - \text{PV(coupons in years 1-3)} \\ &= 1216.50 - 120 / 1.02 - 120 / 1.04^2 - 120 / 1.06^3 = \$887.15. \end{aligned}$$

Solving for the YTM:

$$YTM(4) = \left( \frac{1120}{887.15} \right)^{1/4} - 1 = 6.000\%.$$

Thus, we have computed the zero coupon yield curve as shown.



- 8-30. HMK Enterprises would like to raise \$10 million to invest in capital expenditures. The company plans to issue five-year bonds with a face value of \$1000 and a coupon rate of 6.5% (annual payments). The following table summarizes the yield to maturity for five-year (annualpay) coupon corporate bonds of various ratings:



Rating	AAA	AA	A	BBB	BB
YTM	6.20%	6.30%	6.50%	6.90%	7.50%

- Assuming the bonds will be rated AA, what will the price of the bonds be?
- How much total principal amount of these bonds must HMK issue to raise \$10 million today, assuming the bonds are AA rated? (Because HMK cannot issue a fraction of a bond, assume that all fractions are rounded to the nearest whole number.)
- What must the rating of the bonds be for them to sell at par?
- Suppose that when the bonds are issued, the price of each bond is \$959.54. What is the likely rating of the bonds? Are they junk bonds?

a. The price will be

$$P = \frac{65}{(1+.063)} + \dots + \frac{65+1000}{(1+.063)^5} = \$1008.36.$$

- b. Each bond will raise \$1008.36, so the firm must issue:  $\frac{\$10,000,000}{\$1008.36} = 9917.13 \Rightarrow 9918$  bonds.

This will correspond to a principle amount of  $9918 \times \$1000 = \$9,918,000$ .

- For the bonds to sell at par, the coupon must equal the yield. Since the coupon is 6.5%, the yield must also be 6.5%, or A-rated.
- First, compute the yield on these bonds:

$$959.54 = \frac{65}{(1+YTM)} + \dots + \frac{65+1000}{(1+YTM)^5} \Rightarrow YTM = 7.5\%.$$

Given a yield of 7.5%, it is likely these bonds are BB rated. Yes, BB-rated bonds are junk bonds.

## Chapter 9

# Valuing Stocks

- 9-1. Assume Evco, Inc., has a current price of \$50 and will pay a \$2 dividend in one year, and its equity cost of capital is 15%. What price must you expect it to sell for right after paying the dividend in one year in order to justify its current price?**

We can use Eq. (9.1) to solve for the price of the stock in one year given the current price of \$50.00, the \$2 dividend, and the 15% cost of capital.

$$50 = \frac{2 + X}{1.15}$$
$$X = 55.50$$

At a current price of \$50, we can expect Evco stock to sell for \$55.50 immediately after the firm pays the dividend in one year.

- 9-2. Anle Corporation has a current price of \$20, is expected to pay a dividend of \$1 in one year, and its expected price right after paying that dividend is \$22.**
- What is Anle's expected dividend yield?**
  - What is Anle's expected capital gain rate?**
  - What is Anle's equity cost of capital?**

- Div yld =  $1/20 = 5\%$
- Cap gain rate =  $(22-20)/20 = 10\%$
- Equity cost of capital =  $5\% + 10\% = 15\%$

- 9-6. Summit Systems will pay a dividend of \$1.50 this year. If you expect Summit's dividend to grow by 6% per year, what is its price per share if its equity cost of capital is 11%?**

$$P = 1.50 / (11\% - 6\%) = \$30$$

- 9-7. Dorpac Corporation has a dividend yield of 1.5%. Dorpac's equity cost of capital is 8%, and its dividends are expected to grow at a constant rate.**
- What is the expected growth rate of Dorpac's dividends?**
  - What is the expected growth rate of Dorpac's share price?**

- Eq 9.7 implies  $r_E = \text{Div Yld} + g$ , so  $8\% - 1.5\% = g = 6.5\%$ .
- With constant dividend growth, share price is also expected to grow at rate  $g = 6.5\%$  (or we can solve this from Eq 9.2).



- 9-12. Colgate-Palmolive Company has just paid an annual dividend of \$0.96. Analysts are predicting an 11% per year growth rate in earnings over the next five years. After then, Colgate's earnings are expected to grow at the current industry average of 5.2% per year. If Colgate's equity cost of capital is 8.5% per year and its dividend payout ratio remains constant, what price does the dividend-discount model predict Colgate stock should sell for?**

PV of the first 5 dividends:

$$PV_{\text{first 5}} = \frac{0.96}{0.085 - 0.11} \left( 1 - \left( \frac{1.11}{1.085} \right)^5 \right) = 5.14217.$$

PV of the remaining dividends in year 5:

$$PV_{\text{remaining in year 5}} = \frac{0.96 \cdot 1.11^5}{0.085 - 0.052} = 51.5689.$$

Discounting back to the present

$$PV_{\text{remaining}} = \frac{51.5689}{1.085^5} = 34.2957.$$

Thus the price of Colgate is

$$P = PV_{\text{first 5}} + PV_{\text{remaining}} = 39.4378.$$

- 9-15. Suppose Cisco Systems pays no dividends but spent \$5 billion on share repurchases last year. If Cisco's equity cost of capital is 12%, and if the amount spent on repurchases is expected to grow by 8% per year, estimate Cisco's market capitalization. If Cisco has 6 billion shares outstanding, what stock price does this correspond to?**

Total payout next year = 5 billion  $\times$  1.08 = \$5.4 billion

Equity Value = 5.4 / (12% - 8%) = \$135 billion

Share price = 135 / 6 = \$22.50

- 9-17. Benchmark Metrics, Inc. (BMI), an all-equity financed firm, just reported EPS of \$5.00 per share for 2008. Despite the economic downturn, BMI is confident regarding its current investment opportunities. But due to the financial crisis, BMI does not wish to fund these investments externally. The Board has therefore decided to suspend its stock repurchase plan and cut its dividend to \$1 per share (vs. almost \$2 per share in 2007), and retain these funds instead. The firm has just paid the 2008 dividend, and BMI plans to keep its dividend at \$1 per share in 2009 as well. In subsequent years, it expects its growth opportunities to slow, and it will still be able to fund its growth internally with a target 40% dividend payout ratio, and reinitiating its stock repurchase plan for a total payout rate of 60%. (All dividends and repurchases occur at the end of each year.)**  
**Suppose BMI's existing operations will continue to generate the current level of earnings per share in the future. Assume further that the return on new investment is 15%, and that reinvestments will account for all future earnings growth (if any). Finally, assume BMI's equity cost of capital is 10%.**

- a. Estimate BMI's EPS in 2009 and 2010 (before any share repurchases).
- b. What is the value of a share of BMI at the start of 2009?

- a. To calculate earnings growth, we can use the formula:  $g = (\text{retention rate}) \times \text{RONI}$ .

In 2008, BMI retains \$4 of its \$5 in EPS, for a retention rate of 80%, and an earnings growth rate of  $80\% \times 15\% = 12\%$ . Thus,  $\text{EPS}_{2009} = \$5.00 \times (1.12) = \$5.60$ .

In 2009, BMI retains \$4.60 of its \$5.60 in EPS, for a retention rate of 82.14% and an earnings growth rate of  $82.14\% \times 15\% = 12.32\%$ . So,  $EPS_{2010} = \$5.60 \times (1.1232) = \$6.29$ .

- b. From 2010 on, the firm plans to retain 40% of EPS, for a growth rate of  $40\% \times 15\% = 6\%$ .

Total Payouts in 2010 are 60% of EPS, or  $60\% \times \$6.29 = \$3.774$ .

Thus, the value of the stock at the end of 2009 is, given the 6% future growth rate,

$$P_{2009} = \$3.77 / (10\% - 6\%) = \$94.35.$$

Given the \$1 dividend in 2009, we get a share price in 2008 of

$$P_{2008} = (\$1 + 94.35) / 1.10 = \$86.68.$$

- 9-18. Heavy Metal Corporation is expected to generate the following free cash flows over the next five years:**

Year	1	2	3	4	5
FCF (\$ millions)	53	68	78	75	82

After then, the free cash flows are expected to grow at the industry average of 4% per year. Using the discounted free cash flow model and a weighted average cost of capital of 14%:

- a. Estimate the enterprise value of Heavy Metal.  
 b. If Heavy Metal has no excess cash, debt of \$300 million, and 40 million shares outstanding, estimate its share price.

a.  $V(4) = 82 / (14\% - 4\%) = \$820$

$$V(0) = 53 / 1.14 + 68 / 1.14^2 + 78 / 1.14^3 + (75 + 820) / 1.14^4 = \$681$$

b.  $P = (681 + 0 - 300) / 40 = \$9.53$

- 9-22. You notice that PepsiCo has a stock price of \$52.66 and EPS of \$3.20. Its competitor, the Coca-Cola Company, has EPS of \$2.49. Estimate the value of a share of Coca-Cola stock using only this data.**

PepsiCo P/E =  $52.66 / 3.20 = 16.46x$ . Apply to Coca-Cola:  $\$2.49 \times 16.46 = \$40.98$ .

- 9-23. Suppose that in January 2006, Kenneth Cole Productions had EPS of \$1.65 and a book value of equity of \$12.05 per share.**



- a. Using the average P/E multiple in Table 9.1, estimate KCP's share price.  
 b. What range of share prices do you estimate based on the highest and lowest P/E multiples in Table 9.1?  
 c. Using the average price to book value multiple in Table 9.1, estimate KCP's share price.  
 d. What range of share prices do you estimate based on the highest and lowest price to book value multiples in Table 9.1?

a. Share price = Average P/E  $\times$  KCP EPS =  $15.01 \times \$1.65 = \$24.77$

b. Minimum =  $8.66 \times \$1.65 = \$14.29$ , Maximum =  $22.62 \times \$1.65 = \$37.32$

c.  $2.84 \times \$12.05 = \$34.22$

d.  $1.12 \times \$12.05 = \$13.50$ ,  $8.11 \times \$12.05 = \$97.73$

**9-24.** Suppose that in January 2006, Kenneth Cole Productions had sales of \$518 million, EBITDA of \$55.6 million, excess cash of \$100 million, \$3 million of debt, and 21 million shares outstanding.



- Using the average enterprise value to sales multiple in Table 9.1, estimate KCP's share price.
- What range of share prices do you estimate based on the highest and lowest enterprise value to sales multiples in Table 9.1?
- Using the average enterprise value to EBITDA multiple in Table 9.1, estimate KCP's share price.
- What range of share prices do you estimate based on the highest and lowest enterprise value to EBITDA multiples in Table 9.1?

- Estimated enterprise value for KCP = Average EV/Sales  $\times$  KCP Sales =  $1.06 \times \$518$  million = \$549 million. Equity Value = EV – Debt + Cash =  $\$549 - 3 + 100 = \$646$  million. Share price = Equity Value / Shares =  $\$646 / 21 = \$30.77$
- \$16.21 – \$58.64
- Est. enterprise value for KCP = Average EV/EBITDA  $\times$  KCP EBITDA =  $8.49 \times \$55.6$  million = \$472 million. Share Price =  $(\$472 - 3 + 100) / 21 = \$27.10$
- \$22.25 – \$33.08

**9-26.** Consider the following data for the airline industry in early 2009 (EV = enterprise value, BV = book value, NM = not meaningful because divisor is negative). Discuss the challenges of using multiples to value an airline.

Company Name	Market Cap	EV	EV/Sales	EV/EBITDA	EV/EBIT	P/E	P/Book
Delta Air Lines	4,799.6	16,887.6	0.7x	15.0x	NM	NM	NM
AMR Corp.	1,296.5	8,743.5	0.4x	17.5x	NM	NM	NM
JetBlue Airways	1,246.9	3,834.9	1.1x	10.4x	25.7x	NM	1.0x
Continental Airlines	1,216.8	4,506.8	0.3x	14.7x	NM	NM	NM
UAL Corp.	701.0	6,192.0	0.3x	NM	NM	NM	NM
Air Tran Holdings	651.3	1,354.7	0.5x	21.7x	NM	NM	2.3x
SkyWest	588.7	1,699.7	0.5x	3.8x	7.5x	6.5x	0.5x
Hawaiian	257.1	262.1	0.2x	1.7x	2.7x	3.6x	NM
Pinnacle Airlines	44.0	699.7	0.8x	6.6x	10.1x	3.4x	1.0x

Source: Capital IQ

All the multiples show a great deal of variation across firms. This makes the use of multiples problematic because there is clearly more to valuation than the multiples reveal. Without a clear understanding of what drives the differences in multiples across airlines, it is unclear what the “correct” multiple to use is when trying to value a new airline.