Transportation Problem (TP)

Aim

To identify, formulate and solve transportation problems within the *Solver*/*Excel*.

Definition and Properties

The **Transportation Problem (TP)** requires the distribution of any homogeneous commodity from any group of m distribution centres – **the origins or sources** – to any group of n receiving centres – **the destinations or sinks** –, in such a way as to minimize the total distribution cost.

Parameters of the Model:

m – number of sources (where the commodity exists);

s_i ($i=1,\ldots,m$) – ***supply*** (units supplied) of the commodity at i;

n – number of destinations (where the commodity is requested);

d_j ($j=1,\ldots,n$) – ***demand*** (units of demand) of the commodity at j;

c_{ij} ($i=1,\ldots,m$; $j=1,\ldots,n$) – unitary transportation cost from origin i to destination j.

Let us assume that the total supply equals the total demand, i.e. the TP is **balanced**:

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$

Considering the amount to ship from the origin i (i=1,...,m) to the destination j (j=1,..,n) defined by x_{ij} , a linear programming formulation follows:

$$Min\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to:

$$\begin{cases} \sum_{j=1}^{n} x_{ij} = s_i & i = 1,\ldots,m \\ \sum_{i=1}^{m} x_{ij} = d_j & j = 1,\ldots,n \\ x_{ij} \geq 0 \quad i = 1,\ldots,m;\ j = 1,\ldots,n \end{cases}$$

The first set of constraints ($i=1,\ldots,m$) impose that the commodity send from each origin i to all the destinations equals the supply at the source (s_i). A condition for each destination is written with the second set ($j=1,\ldots,n$). Here it is guaranteed that the units of the commodity that arrive to each destination, from all the sources, equal its demand (d_j). The third set of constraints establishes the set of valid values for the variables.

Transportation Properties

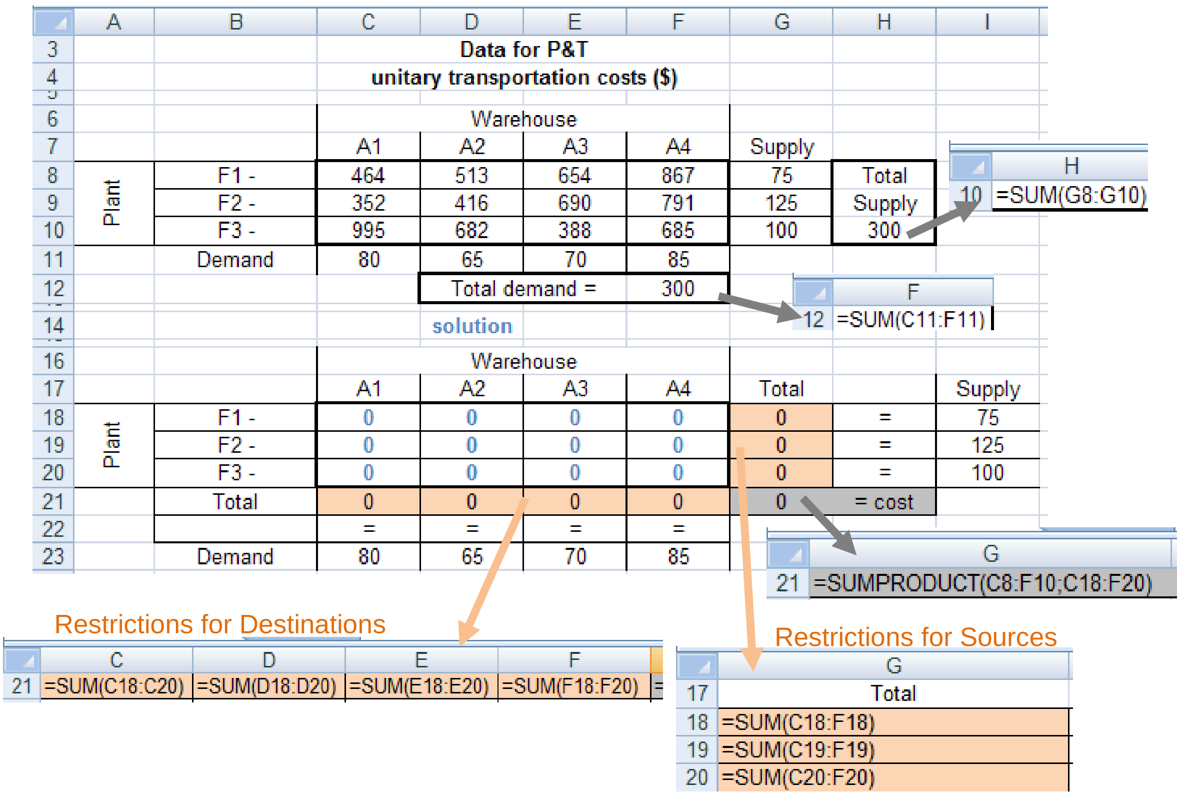
Property 1: A TP has at least a feasible solution.

Corollary: A TP has an optimal solution.

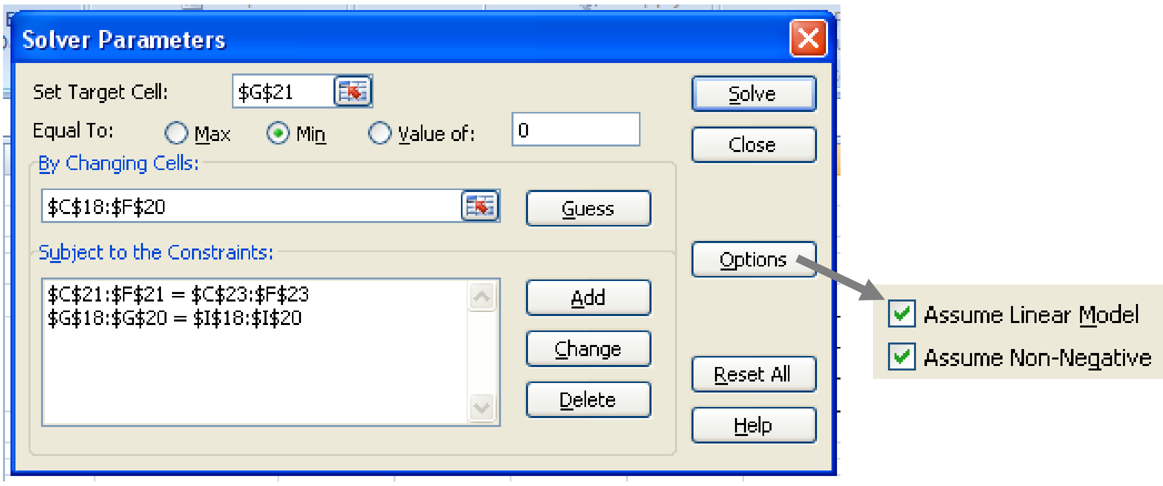
Property 2: A TP with integer supplies and demands has at least one integer optimal solution.

Resolution of the TP with the *Solver*/*Excel*

Prototype example – *P&T* – the problem must be defined in an Excel spreadsheet using two tables. The first table stands for the data: unitary costs (C8:F10), supplies (G8:G10) and the demands (C11:F11). At the second table one must write the problem solution (C18:F20), initially null, and all the formulas needed to establish the restrictions (G18:G20 for the sources, and C21:F21 for the destinations) and the objective function (G21). The relationship between the total supply and the total demand is written within formulas at cells H10 and F12.



Solver – Indentify the target cell (G21), the objective ("Minimize" or "Maximize") and the cells for the values of the variables (C18:F20). Definition of the functional constraints ("Add") of the model (G18:G20=I18:I20 and C21:F21=C23:F23). The problem solution may then be found ("Solve").



Solution – Interpretation of the solution, using all the information provided by the *Excel* sheet or by the answer report.

	B	C	D	E	F	G	H	I
14			solution					
16			Warehouse					
17		A1	A2	A3	A4	Total		Supply
18	F1 -	0	20	0	55	75	=	75
19	F2 -	80	45	0	0	125	=	125
20	F3 -	0	0	70	30	100	=	100
21	Total	80	65	70	85	152535	= cost	
22		=	=	=	=			
23	Demand	80	65	70	85			

	A	B	C	D	E
1	Microsoft Excel 12.0 Answer Report				
2					
3	Target Cell (Min)				
4		Cell	Name	Original Value	Final Value
5		G21	Total Total	0	152535
6					
7	Adjustable Cells				
8		Cell	Name	Original Value	Final Value
9		C18	F1 - A1	0	0
10		D18	F1 - A2	0	20
11		E18	F1 - A3	0	0
12		F18	F1 - A4	0	55
13		C19	F2 - A1	0	80
14		D19	F2 - A2	0	45
15		E19	F2 - A3	0	0
16		F19	F2 - A4	0	0
17		C20	F3 - A1	0	0
18		D20	F3 - A2	0	0
19		E20	F3 - A3	0	70
20		F20	F3 - A4	0	30

Answer: From **F1** send 20 trucks to **A2** and 55 to **A4**; send 80 trucks from **F2** to **A1** and 45 to **A2**; from **F3** send 70 to **A3** and 30 to **A4**. The total cost for this transportation plan is 152’535$.

Variants of the Transportation Problem

(1) Total Supply > Total Demand

The supply of each origin represents a maximum. Then, constraints for the origins should be of type "$\leq$".

(2) Total Supply < Total Demand

The demand in each destination represents a maximum. Then, constraints for the destinations should be of type "$\leq$".

(3) A destination with maximum and minimum demand

If any value between the minimum and the maximum may be shipped, the destination must have 2 constraints: one "$\leq$" and another "$\geq$".

(4) An origin with maximum and minimum supply – similar to (3).

(5) Unfeasible linking

If the shipment between a source *i* and a destination *j* is not feasible, the variable must be set equal to zero, i.e., $x_{ij} = 0$.

Assignment Problem (AP)

Aim

To identify, formulate and solve assignment problems within the *Solver*/*Excel*.

Assignment Problem (AP) – how to assign ***n*** people to ***n*** tasks, minimizing the total assignment cost.

Parameters of the Model:

n – number of assignees and tasks;

c_{ij} ($i=1,\ldots,m$; $j=1,\ldots,n$) – cost of assign person i to job j.

Defining $x_{ij} = \begin{cases} 1 & \text{if assignee } i \text{ performs task } j \\ 0 & \text{if not} \end{cases}$ with $i, j = 1,\ldots,n$, the linear programming model for the AP is:

$$Min\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$ Minimization of the total assignment cost

$$\text{s.a:} \begin{cases} \sum_{j=1}^{n} x_{ij} = 1 & i = 1,\ldots,n \\ \sum_{i=1}^{m} x_{ij} = 1 & j = 1,\ldots,n \\ x_{ij} \in \{0,1\} & i, j = 1,\ldots,n \end{cases}$$

each assignee performs exactly one task

each task is performed by exactly one assignee

binary variables.

Binary variables conditions may be replaced by non-negativity, $x_{ij} \geq 0$ $(i, j = 1,\ldots,n)$, and the model becomes a special type of a linear programming transportation problem. The AP is then a special case of a TP, with unitary supplies and demands, and the same number of sources and destinations.

An AP may be solved by the *Solver* software, using the process explained for the TP. Similarly, the TP variants may also be considered for the AP.