Lecture 7

Varian, Chs. 17.6 to 17.9 and 18

1 Summary of Lectures 1, 2, and 3: Production theory

2 Summary of Lectures 4 and 5: Consumption theory

3 Summary of Lecture 6: Exchange

3.1 Walrasian equilibrium

The solution to a consumer's utility maximization problem

$$\begin{aligned} & \underset{x_i}{Max}\ u(x_i) \\ s.t.\ p \cdot x_i &= p \cdot \omega_i \end{aligned}$$

when taking prices as given is the consumer's demand function. In equilibrium aggregate demand cannot exceed endowments; thus, a Walrasian equilibrium is a pair (p^*, x^*) such that:

$$\sum_i x_i(p^*, p^* \cdot \omega_i) \leq \sum_i \omega_i.$$

If all goods are "desirable", demand equals suppply in all markets.

The aggregate excess demand function is:

$$z(p) = \sum_i [x_i(p, p \cdot \omega_i) - \omega_i].$$

3.2 Edgeworth box (2 consumer case)

3.3 Existence of Walrasian equilibria

$z(p)$ satisfies:

1. homogeneity of degree zero in prices
2. continuity (when all individual demand functions are continuous)
3. Walra's law: for any p, $p \cdot z(p) \equiv 0$.

Proposition 1 *If we know that all markets but market k clears and $p_k > 0$, then market k must also clear.*

Proposition 2 *If a good is in excess supply in a Walrasian equilibrium, i.e., $z_j(p^*) < 0$, it must be a free good: $p^*_j = 0$.*

Proposition 3 *If all goods are desirable and p^* is a Walrasian equilibrium, then $z^*(p) = 0$.*

Definition 4 *Walrasian equilibrium: (x^*, p^*) is a Walrasian equilibrium iff (i) the allocation is feasible $\sum\limits_i x^*_i = \sum\limits_i \omega_i$ and (ii) each agent makes an optimal choice: if x'_i is preferred to x_i, then $px'_i > p\omega_i$.*

Proposition 5 *If $z : S^{k-1} \to \Re^k$ is a continuous function that satisfies Walras' law, $pz(p) \equiv 0$, then there exists some p^* such that $z(p^*) \leq 0$.*

3.4 Pareto efficiency

Definition 6 *A feasible allocation x is weakly Pareto efficient if there is no other feasible allocation that everybody strictly prefers to x.*

Definition 7 *A feasible allocation x is strongly Pareto efficient if there is no other feasible allocation that everybody weakly prefers and some strictly prefer to x.*

Proposition 8 *When preferences are continuous and monotonic, waek and strong Pareto efficiency coincide*

We can illustrate the set of feasible allocations and the potential gains from trade in an Edgeworth box.

The set of Pareto efficient allocations is called the *Pareto set* or *the contract curve.*

3.5 What are the properties of a Walrasian equilibrium?

Theorem 9 First Theorem of Welfare Economics*: If (x, p) is a Walrasian equilibrium, then x is Pareto efficient.*

Agents choose bundles so that $MRS(x_1, x_2) = p_1/p_2$. Since all agents face the same prices they adjust consumption so that their MRS's equal. Hence, in a Walrasian equilibrium agents' indifference curves are tangent and separated by the budget line. The bundles preferred by A are thus separated from those preferred by B which suggests that a Walrasian equilibrium is also Pareto efficient.

The proof uses the definition of Walrasian equilibria which says that if there is another feasible allocation $x\prime$ that is preferred to x by agent i then that allocation must be outside agent i's budget set - otherwise agent i wouldn't have optimized. This is true for all agents and aggregation over the agents' budget constraints tells us that $px\prime > p\omega$, i.e., the allocation $x\prime$is more expensive than the endowment when valued at the same prices. This cannot happen if the amount of each good in the bundle $x\prime$ does not exceed the endowment. Hence, $x\prime$ cannot be feasible.

Theorem 10 Second Theorem of Welfare Economics*: Let x^* be a Pareto efficient allocation in which each agent holds a positive amount of each good. Suppose preferences are convex, continuous, and monotonic. Then, x^* is a Walrasian equilibrium for the initial endowments $\omega_i = x_i^*$, $i = 1, ..., n$.*

The proof starts out with noting that (1) the current Pareto efficient allocation x is feasible and (2) that the set of "endowments", P, that suffice to make all agents strictly better off is a convex set, since the individual "better sets" are convex. The current allocation does not belong to P and the separating hyper-plane theorem says that there exists a price vector p such that $pz \geq px$ for all z in P. It is then shown that $y_i \succ_i x_i$ implies $py_i > px_i$ and thus (x, p) is a Walrasian equilibrium.

Hence, under these conditions any Pareto efficient allocation can be reached via the market mechanism with a suitable reallocation of endowments. Redistribution of wealth is then best done directly rather than by manipulating prices. To tax or subsidize goods distorts the price signals and leads to allocative inefficiencies. Lump sum transfers of endowments allows us to redistribute without sacrificing efficiency. In practice, however, lump sum transfers are often not feasible.

3.6 Welfare maximization

Definition 11 *Social welfare function: $W : \Re^n \to \Re.t$*

We will assume that$W$ is increasing in its arguments.

Proposition 12 *If x^* maximizes a social welfare function, then x^* is Pareto efficient.*

Proposition 13 *Let x^* be a Pareto efficient allocation with $x_i^* \gg 0$, for $i = 1, ..., n$.Let the utility functions u_i be concave, continuous, and monotonic. Then, there is a choice of weights a_i^* such that x^* maximizes $\sum a_i^* u_i(x_i)$ s.t. the resource constraints. Moreover, $a_i^* = \frac{1}{\frac{\partial v_i(p*, m_i)}{\partial m_i}}$.*

4 General equilibrium with production

Consumers ultimately own the firms and consequently this needs to be taken into account in their budget constraints. Specifically, let Tij be consumer i's share of firm j. Consumer i's budget constraint is then:

$$px_i = pw_i + \sum_{j=1}^{m} T_{ij} py_j(p).$$

Aggregate demand is the sum of individual demands and the excess demand function is $z(p) = X(p) - Y(p) - T$. Walras' law holds in the production economy for the same reason as in the exchange economy.

4.1 Existence of Walrasian equilibrium

Existence of an equilibrium now requires specific assumptions about the production technology. Specifically, the aggregate production possibilities set should be convex. While constant returns to scale is a reasonable assumption about technology, the presence of fixed factors may make it impossible to increase scale proportionally for all factors. Increasing the scale of the remaining factors is likely be subject to diminishing returns.

4.2 Pareto efficiency

If a Walrasian equilibrium exists the 1st theorem of welfare economics holds also in this case, i.e. a Walrasian equilibrium is Pareto efficient.

Proposition 14 *The 1st theorem of welfare economics: If there are markets for all commodities which enter into production and utility functions and all markets are competitive, the equilibrium of the economy is Pareto efficient.*

Remark 15 *The proof resembles that for the exchange economy.*

Proposition 16 *The 2nd theorem of welfare economics: In addition to convex consumer preferences the second theorem of welfare economics also requires the firm's production sets to be convex. This rules out increasing returns to scale. Thus, given an appropriate redistribution of endowments, including ownership of firms and labor, all Pareto efficient allocations can be reached as Walrasian equilibria.*

4.3 Welfare