## GESTÃO FINANCEIRA II

## PROBLEM SET 3 - SOLUTIONS

## (FROM BERK AND DEMARZO’S "CORPORATE FINANCE")

## LICENCIATURA - UNDERGRADUATE COURSE

$1^{\text {ST }}$ SEMESTER 2010-2011

## Chapter 10

## Capital Markets and the Pricing of Risk

10-1. The figure below shows the one-year return distribution for RCS stock. Calculate
$X$ a. The expected return.
b. The standard deviation of the return.

a. $\quad E[R]=-0.25(0.1)-0.1(0.2)+0.1(0.25)+0.25(0.3)=5.5 \%$
b.

$$
\begin{aligned}
& \operatorname{Var}[R]=0.1 \times(-0.25-0.055)^{2}+0.2 \times(-0.1-0.055)^{2}+0.15 \times(0-0.055)^{2} \\
& +0.25 \times(0.1-0.055)^{2}+0.3 \times(0.25-0.055)^{2}=0.026
\end{aligned}
$$

Standard Deviation $=\sqrt{0.026}=16.13 \%$

10-4. You bought a stock one year ago for $\$ 50$ per share and sold it today for $\$ 55$ per share. It paid a $\$ 1$ per share dividend today.
a. What was your realized return?
b. How much of the return came from dividend yield and how much came from capital gain? Compute the realized return and dividend yield on this equity investment.
a. $\quad R=\frac{1+(55-50)}{50}=0.12=12 \%$
b. $\quad R_{\text {div }}=\frac{1}{50}=2 \%$

$$
R_{\text {capital gain }}=\frac{55-50}{50}=10 \%
$$

The realized return on the equity investment is $12 \%$. The dividend yield is $10 \%$.

10-6. Using the data in the following table, calculate the return for investing in Boeing stock from January 2, 2003, to January 2, 2004, and also from January 2, 2008, to January 2, 2009, assuming all dividends are reinvested in the stock immediately.

Historical Stock and Dividend Data for Boeing

| Date | Price | Dividend | Date | Price | Dividend |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $1 / 2 / 03$ | 33.88 |  | $1 / 2 / 08$ | 86.62 |  |
| $2 / 5 / 03$ | 30.67 | 0.17 | $2 / 6 / 08$ | 79.91 | 0.40 |
| $5 / 14 / 03$ | 29.49 | 0.17 | $5 / 7 / 08$ | 84.55 | 0.40 |
| $8 / 13 / 03$ | 32.38 | 0.17 | $8 / 6 / 08$ | 65.40 | 0.40 |
| $11 / 12 / 03$ | 39.07 | 0.17 | $11 / 5 / 08$ | 49.55 | 0.40 |
| $1 / 2 / 04$ | 41.99 |  | $1 / 2 / 09$ | 45.25 |  |


| Date | Price | Dividend | $\mathbf{R}$ | $\mathbf{1}+\mathbf{R}$ |
| ---: | :---: | :---: | ---: | :---: |
| $1 / 2 / 2003$ | 33.88 |  |  |  |
| $2 / 5 / 2003$ | 30.67 | 0.17 | $-8.97 \%$ | 0.910272 |
| $5 / 14 / 2003$ | 29.49 | 0.17 | $-3.29 \%$ | 0.967069 |
| $8 / 13 / 2003$ | 32.38 | 0.17 | $10.38 \%$ | 1.103764 |
| $11 / 12 / 2003$ | 39.07 | 0.17 | $21.19 \%$ | 1.211859 |
| $1 / 2 / 2004$ | 41.99 |  | $7.47 \%$ | 1.074738 |
|  |  |  |  |  |
|  |  |  | $26.55 \%$ | 1.265491 |
|  |  |  |  |  |
|  |  |  |  |  |
| Date | Price | Dividend | $\mathbf{R}$ | $\mathbf{1}+\mathbf{R}$ |
| $1 / 2 / 2008$ | 86.62 |  |  |  |
| $2 / 6 / 2008$ | 79.91 | 0.4 | $-7.28 \%$ | 0.927153 |
| $5 / 7 / 2008$ | 84.55 | 0.4 | $6.31 \%$ | 1.063071 |
| $8 / 6 / 2008$ | 65.4 | 0.4 | $-22.18 \%$ | 0.778238 |
| $11 / 5 / 2008$ | 49.55 | 0.4 | $-23.62 \%$ | 0.763761 |
| $1 / 2 / 2009$ | 45.25 |  | $-8.68 \%$ | 0.913219 |
|  |  |  |  |  |
|  |  |  | $-46.50 \%$ | 0.535006 |
|  |  |  |  |  |

10-9. Consider an investment with the following returns over four years:

| 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: |
| $10 \%$ | $20 \%$ | $-5 \%$ | $15 \%$ |

a. What is the compound annual growth rate (CAGR) for this investment over the four years?
b. What is the average annual return of the investment over the four years?
c. Which is a better measure of the investment's past performance?
d. If the investment's returns are independent and identically distributed, which is a better measure of the investment's expected return next year?
a.

| $\underline{\mathbf{1}}$ | $\underline{\mathbf{2}}$ | $\underline{\mathbf{3}}$ | $\underline{4}$ |  | Ave |
| ---: | ---: | ---: | ---: | ---: | :---: |
| $10 \%$ | $20 \%$ | $-5 \%$ | $15 \%$ |  | $10.00 \%$ |
|  |  |  |  |  | CAGR |
| 1.10 | 1.20 | 0.95 | 1.15 |  | $9.58 \%$ |

b. see table above
c. CAGR
d. Arithmetic average

10-15. Download the spreadsheet from MyFinanceLab containing the data for Figure 10.1.
a. Compute the average return for each of the assets from 1929 to 1940 (The Great Depression).
b. Compute the variance and standard deviation for each of the assets from 1929 to 1940.
c. Which asset was riskiest during the Great Depression? How does that fit with your intuition?
$\mathrm{a} / \mathrm{b}$.

|  | S\&P |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | 5 |  |  |  |  |  |
|  |  | 0 |  |  |  |  |  |
|  |  | 0 | SmallStocks | CorpBonds | WorldPortfolio | TreasuryBills | CPI |
| Average | $2.553 \%$ | $16.550 \%$ | $5.351 \%$ | $2.940 \%$ | $0.859 \%$ | $1.491 \%$ |  |
| Variance: | 0.1018 | 0.6115 | 0.0013 | 0.0697 | 0.0002 | 0.0022 |  |
| Standarddeviation: | $31.904 \%$ | $78.195 \%$ | $3.589 \%$ | $26.398 \%$ | $1.310 \%$ | $4.644 \%$ |  |

Evaluate:
c. The riskiest assets were the small stocks. Intuition tells us that this asset class would be the riskiest.

10-16. Using the data from Problem 15, repeat your analysis over the 1990s.
a. Which asset was riskiest?
b. Compare the standard deviations of the assets in the 1990s to their standard deviations in the Great Depression. Which had the greatest difference between the two periods?
c. If you only had information about the 1990 s, what would you conclude about the relative risk of investing in small stocks?
a. Using Excel:

|  | S\&P 500 | SmallStocks | Corp Bonds | World Portfolio | TreasuryBills | CPI |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Average | $18.990 \%$ | $14.482 \%$ | $9.229 \%$ | $12.819 \%$ | $4.961 \%$ | $2.935 \%$ |
| Variance: | 0.0201 | 0.0460 | 0.0062 | 0.0194 | 0.0002 | 0.0002 |
| Standard <br> deviation: | $14.161 \%$ | $21.451 \%$ | $7.858 \%$ | $13.938 \%$ | $1.267 \%$ | $1.239 \%$ |

The riskiest asset class was small stocks.
b. The greatest absolute difference in standard deviation is in the small stocks asset class, which saw standard deviation fall $56.7 \%$. But in relative terms, the riskiness of corporate bonds rose $118 \%$ (relative to 1940), while the riskiness of small stocks fell only $72.6 \%$ (relative to 1940 levels). Inflation is now much less risky as well, falling in relative riskiness by $73.3 \%$.
c. If you were only looking at the 1990 s, you would conclude that small stocks are relatively less risky than they actually are.
The results that one can derive from analyzing data from a particular time period can change depending on the time period analyzed. These differences can be large if the time periods being analyzed are short.

## Chapter 11

## Optimal Portfolio Choice and the Capital Asset Pricing Model

11-1. You are considering how to invest part of your retirement savings. You have decided to put $\mathbf{\$ 2 0 0 , 0 0 0}$ into three stocks: $\mathbf{5 0 \%}$ of the money in GoldFinger (currently $\mathbf{\$ 2 5} /$ share), $\mathbf{2 5 \%}$ of the money in Moosehead (currently \$80/share), and the remainder in Venture Associates (currently $\$ \mathbf{2} /$ share). If GoldFinger stock goes up to $\$ 30 /$ share, Moosehead stock drops to $\$ 60 /$ share, and Venture Associates stock rises to $\$ 3$ per share,
a. What is the new value of the portfolio?
b. What return did the portfolio earn?
c. If you don't buy or sell shares after the price change, what are your new portfolio weights?
a. Let ${ }^{n_{i}}$ be the number of share in stock I, then
$n_{G}=\frac{200,000 \times 0.5}{25}=4,000$
$n_{M}=\frac{200,000 \times 0.25}{80}=625$
$n_{V}=\frac{200,000 \times 0.25}{2}=25,000$.
The new value of the portfolio is
$p=30 n_{G}+60 n_{M}+3 n_{v}$
$=\$ 232,500$.
b. Return $=\frac{232,500}{200,000}-1=16.25 \%$
c. The portfolio weights are the fraction of value invested in each stock.

GoldFinger: $\frac{n_{G} \times 30}{232,500}=51.61 \%$
Moosehead: $\frac{n_{M} \times 60}{232,500}=16.13 \%$
Venture: $\frac{n_{V} \times 3}{232,500}=32.26 \%$
11-2. You own three stocks: 1000 shares of Apple Computer, 10,000 shares of Cisco Systems, and 5000 shares of Goldman Sachs Group. The current share prices and expected returns of Apple, Cisco, and Goldman are, respectively, \$125, \$19, \$120 and $\mathbf{1 2 \%}$, 10\%, 10.5\%.
a. What are the portfolio weights of the three stocks in your portfolio?
b. What is the expected return of your portfolio?
c. Assume that both Apple and Cisco go up by $\$ 5$ and Goldman goes down by $\$ 10$. What are the new portfolio weights?
d. Assuming the stocks' expected returns remain the same, what is the expected return of the portfolio at the new prices?
$\begin{array}{llll}\text { Apple } & 1000 & 125 & 12\end{array}$

| Value | a. | b. | New Price | New Value | c. | d. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 125000 | 0.136612022 | 1.639344262 | 130 | 130000 | 0.14207650 | 1.704918033 |

$\begin{array}{llll}\text { Cisco } & 10000 & 19 & 10\end{array}$

| Value | a. | b. | New Price | New Value |  | c. |
| :--- | :---: | :---: | ---: | :--- | :---: | :---: |$c$ d..


| Goldman | 5000 | 120 | 10.5 |
| :--- | :--- | :--- | :--- |


| Value | a. | b. | New Price | New Value | c. | d. |
| :--- | :--- | :--- | :--- | :---: | :---: | :---: |
| 600000 | 0.655737705 | 6.885245902 | 110 | 550000 | 0.601092896 | 6.31147541 |

Total
915000
10.6010929
10.63934426

11-5. Using the data in the following table, estimate (a) the average return and volatility for each stock, (b) the covariance between the stocks, and (c) the correlation between these two stocks.

| Year | $\mathbf{2 0 0 4}$ | $\mathbf{2 0 0 5}$ | $\mathbf{2 0 0 6}$ | $\mathbf{2 0 0 7}$ | $\mathbf{2 0 0 8}$ | $\mathbf{2 0 0 9}$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Stock A | $-10 \%$ | $20 \%$ | $5 \%$ | $-5 \%$ | $2 \%$ | $9 \%$ |
| Stock B | $21 \%$ | $7 \%$ | $30 \%$ | $-3 \%$ | $-8 \%$ | $25 \%$ |

a. $\quad \bar{R}_{A}=\frac{-10+20+5-5+2+9}{6}=3.5 \%$

$$
\begin{aligned}
\bar{R}_{B} & =\frac{21+30+7-3-8+25}{6} \\
& =12 \%
\end{aligned}
$$

$$
\begin{aligned}
\text { Variance of } \mathrm{A} & =\frac{1}{5}\left[\begin{array}{l}
(-0.1-0.035)^{2}+ \\
(0.2-0.08)^{2}+(0.05-0.035)^{2}+ \\
(-0.05-0.035)^{2}+(0.02-0.035)^{2} \\
+(0.09-0.035)^{2}
\end{array}\right] \\
& =0.01123
\end{aligned}
$$

Volatility of $\mathrm{A}=S D\left(R_{A}\right)=\sqrt{\text { Variance of } \mathrm{A}}=\sqrt{.01123}=10.60 \%$

Variance of $B=\frac{1}{5}\left[\begin{array}{l}(0.21-0.12)^{2}+(0.3-0.12)^{2}+ \\ (0.07-0.12)^{2}+(-0.03-0.12)^{2}+ \\ (-0.08-0.12)^{2}+(0.25-0.12)^{2}\end{array}\right]$
$=0.02448$
Volatility of B $=S D\left(R_{B}\right)=\sqrt{\text { Variance of } \mathrm{B}}=\sqrt{.02448}=15.65 \%$
b. Covariance $=\frac{1}{5}\left[\begin{array}{l}(-0.1-0.035)(0.21-0.12)+ \\ (0.2-0.035)(0.3-0.12)+ \\ (0.05-0.035)(0.07-0.12)+ \\ (-0.05-0.035)(-0.03-0.12)+ \\ (0.02-0.035)(-0.08-0.12)+ \\ (0.09-0.035)(0.25-0.12)\end{array}\right]$

$$
=0.104 \%
$$

c. Correlation $=\frac{\text { Covariance }}{\operatorname{SD}\left(R_{A}\right) \operatorname{SD}\left(R_{B}\right)}$

$$
=6.27 \%
$$

11-6. Use the data in Problem 5, consider a portfolio that maintains a $50 \%$ weight on stock $A$ and a $\mathbf{5 0 \%}$ weight on stock B.
a. What is the return each year of this portfolio?
b. Based on your results from part a, compute the average return and volatility of the portfolio.
c. Show that (i) the average return of the portfolio is equal to the average of the average returns of the two stocks, and (ii) the volatility of the portfolio equals the same result as from the calculation in Eq. 11.9.
d. Explain why the portfolio has a lower volatility than the average volatility of the two stocks. a, b, and c. See table below.

| Year | $\mathbf{2 0 0 4}$ | $\mathbf{2 0 0 5}$ | $\mathbf{2 0 0 6}$ | $\mathbf{2 0 0 7}$ | $\mathbf{2 0 0 8}$ | $\mathbf{2 0 0 9}$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| A\&B | $5.5 \%$ | $13.5 \%$ | $17.5 \%$ | $-4.0 \%$ | $-3.0 \%$ | $17.0 \%$ |
| Ave | $7.75 \%$ |  |  |  |  |  |
| Vol | $9.72 \%$ |  |  |  |  |  |

d. The portfolio has a lower volatility than the average volatility of the two stocks because some of the idiosyncratic risk of the stocks in the portfolio is diversified away.

For Problems 22-25, suppose Johnson \& Johnson and the Walgreen Company have expected returns and volatilities shown below, with a correlation of $22 \%$.

|  | $E[R]$ | $S D[R]$ |
| :--- | :---: | :---: |
| Johnson \& Johnson | $7 \%$ | $16 \%$ |
| Walgreen Company | $10 \%$ | $20 \%$ |

11-22. Calculate (a) the expected return and (b) the volatility (standard deviation) of a portfolio that is equally invested in Johnson \& Johnson's and Walgreen's stock.

In this case, the portfolio weights are $\mathrm{x}_{\mathrm{j}} \mathrm{x}_{\mathrm{w}}=0.50$. From Eq. 11.3,

$$
\begin{aligned}
E\left[R_{P}\right] & =x_{j} E\left[R_{j}\right]+x_{w} E\left[R_{w}\right] \\
& =0.50(7 \%)+0.50(10 \%) \\
& =8.5 \% .
\end{aligned}
$$

We can use Eq. 11.9.

$$
\begin{aligned}
\operatorname{SD}\left(R_{P}\right) & =\sqrt{x_{j}^{2} S D\left(R_{j}\right)^{2}+x_{w}^{2} \operatorname{SD}\left(R_{w}\right)^{2}+2 x_{j} x_{w} \operatorname{Corr}\left(R_{j}, R_{w}\right) \operatorname{SD}\left(R_{j}\right) \operatorname{SD}\left(R_{w}\right)} \\
& =\sqrt{.50^{2}\left(.16^{2}\right)+.50^{2}(.20)^{2}+2(.50)(.50)(.22)(.16)(.20)} \\
& =14.1 \%
\end{aligned}
$$

11-23. For the portfolio in Problem 22, if the correlation between Johnson \& Johnson's and Walgreen's stock were to increase,
a. Would the expected return of the portfolio rise or fall?
b. Would the volatility of the portfolio rise or fall?
a. The expected return would remain constant, assuming only the correlation changes, $0.5 \times 0.07+$ $0.5 \times 0.10=0.085$.
b. The volatility of the portfolio would increase (due to the correlation term in the equation for the volatility of a portfolio).

11-24. Calculate (a) the expected return and (b) the volatility (standard deviation) of a portfolio that consists of a long position of $\$ 10,000$ in Johnson \& Johnson and a short position of $\$ 2000$ in Walgreen's.
In this case, the total investment is $\$ 10,000-2,000=\$ 8,000$, so the portfolio weights are $\mathrm{x}_{\mathrm{j}}=10,000 / 8,000=$ $1.25, \mathrm{x}_{\mathrm{w}}=-2,000 / 8,000=-0.25$. From Eq. 11.3,

$$
\begin{aligned}
E\left[R_{P}\right] & =x_{j} E\left[R_{j}\right]+x_{w} E\left[R_{w}\right] \\
& =1.25(7 \%)-0.25(10 \%) \\
& =6.25 \% .
\end{aligned}
$$

We can use Eq. 11.9,

$$
\begin{aligned}
\operatorname{SD}\left(R_{P}\right) & =\sqrt{x_{j}^{2} S D\left(R_{j}\right)^{2}+x_{w}^{2} \operatorname{SD}\left(R_{w}\right)^{2}+2 x_{j} x_{w} \operatorname{Corr}\left(R_{j}, R_{w}\right) \operatorname{SD}\left(R_{j}\right) \operatorname{SD}\left(R_{w}\right)} \\
& =\sqrt{1.25^{2}\left(.16^{2}\right)+(-0.25)^{2}(.20)^{2}+2(1.25)(-0.25)(.22)(.16)(.20)} \\
& =19.5 \% .
\end{aligned}
$$

11-25. Using the same data as for Problem 22, calculate the expected return and the volatility (standard deviation) of a portfolio consisting of Johnson \& Johnson's and Walgreen's stocks using a wide range of portfolio weights. Plot the expected return as a function of the portfolio volatility. Using your graph, identify the range of Johnson \& Johnson's portfolio weights that yield efficient combinations of the two stocks, rounded to the nearest percentage point.
The set of efficient portfolios is approximately those portfolios with no more than $65 \%$ invested in J\&J (this is the portfolio with the lowest possible volatility).

| $\mathbf{x}(\mathbf{J} \& \mathbf{J})$ | $\mathbf{x}$ (Walgreen) | SD | ER |
| :---: | :---: | :---: | :---: |
| $-50 \%$ | $150 \%$ | $29.30 \%$ | $11.50 \%$ |
| $-40 \%$ | $140 \%$ | $27.32 \%$ | $11.20 \%$ |
| $-30 \%$ | $130 \%$ | $25.38 \%$ | $10.90 \%$ |
| $-20 \%$ | $120 \%$ | $23.50 \%$ | $10.60 \%$ |
| $-10 \%$ | $110 \%$ | $21.70 \%$ | $10.30 \%$ |
| $0 \%$ | $100 \%$ | $20.00 \%$ | $10.00 \%$ |
| $10 \%$ | $90 \%$ | $18.42 \%$ | $9.70 \%$ |
| $20 \%$ | $80 \%$ | $16.99 \%$ | $9.40 \%$ |
| $30 \%$ | $70 \%$ | $15.77 \%$ | $9.10 \%$ |
| $40 \%$ | $60 \%$ | $14.79 \%$ | $8.80 \%$ |
| $50 \%$ | $50 \%$ | $14.11 \%$ | $8.50 \%$ |
| $60 \%$ | $40 \%$ | $13.78 \%$ | $8.20 \%$ |
| $65 \%$ | $35 \%$ | $13.75 \%$ | $8.05 \%$ |
| $70 \%$ | $30 \%$ | $13.82 \%$ | $7.90 \%$ |
| $80 \%$ | $20 \%$ | $14.23 \%$ | $7.60 \%$ |
| $90 \%$ | $10 \%$ | $14.97 \%$ | $7.30 \%$ |
| $100 \%$ | $0 \%$ | $16.00 \%$ | $7.00 \%$ |
| $110 \%$ | $-10 \%$ | $17.27 \%$ | $6.70 \%$ |
| $120 \%$ | $-20 \%$ | $18.73 \%$ | $6.40 \%$ |
| $130 \%$ | $-30 \%$ | $20.34 \%$ | $6.10 \%$ |
| $140 \%$ | $-40 \%$ | $22.07 \%$ | $5.80 \%$ |
| $150 \%$ | $-50 \%$ | $23.88 \%$ | $5.50 \%$ |

11-47. Consider a portfolio consisting of the following three stocks:

|  | Portfolio Weight | Volatility | Correlation with <br> the Market Portfolio |
| :--- | :---: | :---: | :---: |
| HEC Corp | 0.25 | $12 \%$ | 0.4 |
| Green Midget | 0.35 | $25 \%$ | 0.6 |
| AliveAndWell | 0.4 | $13 \%$ | 0.5 |

The volatility of the market portfolio is $\mathbf{1 0 \%}$ and it has an expected return of $\mathbf{8 \%}$. The risk-free rate is $\mathbf{3 \%}$.
a. Compute the beta and expected return of each stock.
b. Using your answer from part a, calculate the expected return of the portfolio.
c. What is the beta of the portfolio?
d. Using your answer from part c , calculate the expected return of the portfolio and verify that it matches your answer to part $b$.

|  | Portfolio Weight | Volatility | Correlation with <br> the Market <br> Portfolio  | Beta (Part a <br> answer) | Expected Return (Part a answer) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| HEC Corp | 0.25 | 12\% | 0.4 | 0.48 | 5.4 |
| Green Midget | 0.35 | 25\% | 0.6 | 1.5 | 10.5 |
| AliveAndWell | 0.4 | 13\% | 0.5 | 0.65 | 6.25 |
|  |  |  |  | Part c answer: | Part b answer |
|  |  |  | Porfolio | 0.905 | 7.525 |
|  |  |  |  |  | Part d answer |
|  |  | Expected Return calculated from porfolio beti |  |  | 7.525 |

11-48. Suppose Intel stock has a beta of 2.16, whereas Boeing stock has a beta of 0.69. If the risk-free interest rate is $4 \%$ and the expected return of the market portfolio is $10 \%$, what is the expected return of a portfolio that consists of $\mathbf{6 0 \%}$ Intel stock and $\mathbf{4 0 \%}$ Boeing stock, according to the CAPM?
$\beta=(0.6)(2.16)+(0.4)(0.69)=1.572$
$E[R]=4+(1.572)(10-4)=13.432 \%$

