## Prototype Example - CALIFORNIA $\mathcal{M A N U F A C T V R I N G ~}^{\prime}$ COMPANVY (HL ${ }^{1}$, §1.1, pg. 465)

The CALIFORNIA MANUFACTURING COMPANVY is considering expansion by building a new factory either in Los Angeles or San Francisco, or perhaps even in both cities. It also is considering building at most one new warehouse, but the choice of a location is restricted to a city where a new factory is being built. The net present value (total profitability considering the time value of the money) of each of these alternatives is shown in the second column of the table below. The rightmost column gives the capital required (already included in the net present value) for the respective investments, where the total capital available is $\$ 10$ million. The objective is to find the feasible combination of alternatives that maximizes the total net present value.

| Build a | Net Present <br> Value | Capital <br> Requirement |
| :--- | :---: | :---: |
| Factory in LA | $\$ 9$ million | $\$ 6$ million |
| Factory in SF | $\$ 5$ million | $\$ 3$ million |
| Warehouse in LA | $\$ 6$ million | $\$ 5$ million |
| Warehouse in SF | $\$ 4$ million | $\$ 2$ million |

## ILP Exercises

48. TBA Airfines is a small air company, specialized in regional flights. The management is considering an expansion and it has the possibility to buy small or medium size airplanes. Find the best strategy, knowing that at the moment no more than two small airplanes can be bought and that $\$ 100$ millions are available to invest. Consider also the values in the following table:

|  | Small airplane | Medium size <br> airplane |
| :--- | :---: | :---: |
| annual profit per airplane | $\$ 1$ million | $\$ 5$ millions |
| Cost per airplane | $\$ 5$ millions | $\$ 50$ millions |

49. A company has a portfolio with three investment projects with a life time of 4 years. The table presents the cash-flows and the corresponding net present values (NPV) at $10 \%$ year rate:

| Project | year 0 | year 1 | year 2 | year 3 | NPV (m.u.) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{A}$ | -100 | 50 | 300 | -20 | 178 |
| $\mathbf{B}$ | -300 | 100 | 150 | 200 | 65 |
| $\mathbf{C}$ | 0 | -100 | 75 | 200 | 121 |

Project $\mathbf{B}$ can be deferred by one year in that case the NPV is $59 \mathrm{~m} . u_{\text {., as }}$, it can be verified. There are budget constraints in the first two years of 300 and 200 m . u., respectively. Formulate this problem and solve it by Solver.
50. An oil company intends to select 5 out of 10 wells: $\mathbf{P 1}, \mathbf{P} 2, \ldots, \mathbf{P 1 0}$, to which are associated the costs $c_{1}, c_{2}, \ldots, c_{10}$, respectively. According to commitments with the local government, the company must comply with the following restrictions for regional development:
r1) the selection of both P1 and P7 block selection of P8;
r2) the selection of $\mathbf{P 3}$ or $\mathbf{P 4}$ block selection of $\mathbf{P 5}$;
r3) from P5, P6, P7 and P8 at most two can be selected;
r4) the selection of $\mathbf{P 1}$ forces selection of $\mathbf{P 1 0}$.
Formulate the problem and solve it assigning costs at your choice.
51. Solve problems 11.1-2; 11.1-3., 11.3-1., 11.4-3 of $\mathrm{HL}^{1}$. (pg. 524-527)
52. ( $\mathrm{HL}^{1}$, pg. 55) Union Airways is adding more flights to and from its hub airport, and so it needs to hire additional customer service agents. However, it is not clear how many more should be hired. Management recognizes the need for cost control while also consistently providing a satisfactory level of service to customers. Therefore, the OR team is studying how to schedule the agents to provide satisfactory service with the smallest personnel cost.
Based on the new schedule of flights an analysis has been made of the minimum number of customer service agents that need to be on duty at different times of the day to provide a satisfactory level of service.

|  | Time Periods Covered |  |  |  |  | Minimum number of Agents Needed |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Sh |  |  |  |
| Time Period | 1 | 2 | 3 | 4 | 5 |  |
| 6:00-8:00 | $\checkmark$ |  |  |  |  | 48 |
| 8:00-10:00 | $\checkmark$ | $\checkmark$ |  |  |  | 79 |
| 10:00-12:00 | $\checkmark$ | $\checkmark$ |  |  |  | 65 |
| 12:00-14:00 | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  | 87 |
| 14:00-16:00 |  | $\checkmark$ | $\checkmark$ |  |  | 64 |
| 16:00-18:00 |  |  | $\checkmark$ | $\checkmark$ |  | 73 |
| 18:00-20:00 |  |  | $\checkmark$ | $\checkmark$ |  | 82 |
| 20:00-22:00 |  |  |  | $\checkmark$ |  | 43 |
| 22:00-24:00 |  |  |  | $\checkmark$ | $\checkmark$ | 52 |
| 0:00-6:00 |  |  |  |  | $\checkmark$ | 15 |
| Daily cost per agent | \$170 | \$160 | \$175 | \$180 | \$195 |  |

Each agent works an 8 -hour shift, five days per week. Determine how many agents should be assigned to the respective shifts each day to minimize the total personnel cost for agents while meeting the service requirements.
53. A company is preparing its investment plan for the next three years. From the current activities predicts the availability of the following funds (in m.u.): 50 in year $1 ; 40$ in year $2 ; 30$ in year 3 . Beyond this self-financing, the company may seek loans from short-term (one year) in each of three years. The amount requested in each year shall be paid the following year along with interest, which is $6 \%$ per year. The company can also get in the 1st year a medium term loan to be paid in four years (at the end of the planning period), and each year the interest, which is $7 \%$ per year. For financial reasons, the company cannot have for both short-term loans in years 1 and 2 . The funds available, including any loans can be applied in the following investment projects:

|  | cash-flow |  |  | TPV* |
| :--- | :---: | :---: | :---: | :---: |
|  | year 1 | year 2 | year 3 | (end of year 3) |
| project 1 | -100 | 20 | 30 | 100 |
| project 2 | -50 | -40 | 50 | 90 |
| project 3 |  | -50 | 50 | 20 |

${ }^{*}$ TPV - Terminal Present Value, that is, the present value at the end of year 3 of the cash-flows that will occur in the 4 coming years.
The company intends to maximize the net assets of liabilities at end of planning period, that is, intends to maximize the TPV projects less debt at the end of the planning period. Knowing that the projects can only be done by all, establish a model for this problem

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[^0]:    ${ }^{1}$ Hillier, Lieberman, "Introduction to Operations Research", $9^{a}$ ed., McGraw-Hill 2010

