Instituto Superior de Economia e Gestão

UNIVERSIDADE TÉCNICA DE LISBOA

Master in Actuarial Science
Examination: Risk Theory
July 1st, 2009
10 a.m. -13 a.m.

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Total number of points: 200

1. In a group disability income insurance, the expected number of disabilities per year is 1 per 100 lives covered. The length of the disability, in days, is a discrete random variable, Y, taking integer values and such that

$$\Pr\{Y > y\} = \ 1 - \tfrac{y}{30} \qquad y = 0, 1, ..., 30.$$

The benefit is 100 m.u. per day of disability. Consider that the Poisson distribution describes well the number of disabilities per year. Consider that for each life there is independence among the disability periods. For a group of 1000 independent lives:

   (a) verify that the expected value, standard deviation and skewness coefficient of the aggregate annual claims are 15 500, 5614. 0 and 0.407 36 respectively.[1] (20)

   (b) calculate an approximation to the probability that the aggregate claims exceed 22500 m.u. (20)

   (c) calculate the exact probability that the aggregate claims are greater than 400 m.u. (20)

2. Let the aggregate claims of a portfolio follow a compound Poisson process, where the individual claims amount, in thousand of euros, has a Gamma distribution with mean 4 and variance 8. The premium loading is 30% of the pure premium.

   (a) Verify that the ultimate probability of ruin, as a function of the initial (25)

[1] Note that

$$\sum_{k=1}^{m} k^2 = \frac{m(1+m)(1+2m)}{6} \ \text{e} \ \sum_{k=1}^{m} k^3 = \frac{m^2(1+m)^2}{4}$$

reserve u, is (note that the irrational numbers were approximated)

$$\psi(u) = 0.794707 \exp(-0.079193u) - 0.025472 \exp(-0.728499u).$$

(b) What is the adjustment coefficient? Considering an initial reserve equal to 50 thousand euros, calculate the upper bound provided by Lundberg's inequality and compare it with the probability of ultimate ruin.

3. Let the severity X (in a given monetary unit) associated with an accident be modelled by a Pareto distribution, i.e.

$$\Pr(X \leq x) = 1 - \left(\frac{\beta}{\beta + x}\right)^{\alpha}, \ x > 0,$$

with mean 1 and variance 3. An insurer company writes policies with a deductible 0.05 per accident, in such a way that the policyholder only claims policies with a severity greater than 0.05 and the insurer only pays the excess over 0.05. Hence, let $Y = \min(X; 0.05)$ and $Z = (X - 0.05)_+ = \max(0; X - 0.05)$ be the cost of an accident taken by the policyholder and by the insurer respectively, when an accident happens.

(a) Prove that the amount paid by the insurer for each **claim** is still Pareto distributed and indicate the distribution parameters. (20)

(b) Calculate $E[Z|Z > 0]$, $E[Z]$ and $E[Y]$. (15)

(c) Let the number of accidents occur according to a Poisson process at a rate of 100 per year. Consider that the insurer calculates the premiums using the expected value principle with a loading coefficient 0.1. The insurer has a quota-share reinsurance treaty, with retention level a, $0 < a < 1$, and where the premium is calculated using the expected value principle with loading coefficient 0.25. Determine the annual net expected profit as a function of a.

(d) Does the adjustment coefficient for the retained claims exist, for some value of a, $0 < a < 1$? Justify your answer. (10)

4. Let X, be a r.v. Pareto distributed with parameters α and β, i.e.

$$\Pr(X \leq x) = 1 - \left(\frac{\beta}{\beta + x}\right)^{\alpha}, \ x > 0.$$

(a) Obtain the Value-at-Risk of X at the $100p\%$ security level. (10)

(b) Obtain the Conditional-Tail-Expectation of X at the $100p\%$ security level. (20)

5. Let the individual claim amounts be distributed according to an exponential distribution with mean 1 (for a given monetary unit). Discretize the distribution by the method of rounding, using a step of size h. Prove that the distribution found is a geometric modified at the origin (change the monetary unit if necessary). (15)